# Math S-21b: Linear Algebra and Differential Equations – Summer 2019

**Instructor**: **Robert Winters** robert@math.rwinters.com or rwinters@mit.edu.

**Teaching Assistant**: **Jeremy Marcq** **Course website**: **http://math.rwinters.com/S21b**

**Class meetings**: Mondays, Wednesdays, and Fridays, 9:30am to 11:30am in **Emerson Hall 210**, Mon, June 24 through Fri, August 2. The week of Aug 5 – Aug 9 will be taken up with reviews and the Final Exam. There will be an optional weekly problem session at a time and location to be determined after the first class meeting.

**Office Hours**: Mondays, Wednesdays, and Fridays immediately following class or elsewhere on campus by other arrangements (office space has not yet been assigned). Other times may be scheduled by appointment.

**Prerequisites**: Math 1b or equivalent knowledge of algebra and calculus; and basic knowledge of vectors and vector algebra (as may be learned in Math 21a or equivalent). You should be able to solve simple systems of equations, find the roots of polynomials, and ideally be able to set up and solve the kind of simple differential equations found in a typical first-year calculus course. Math 21a or an equivalent multivariable calculus course is not necessary for successful completion of this course, though it is preferable. Math 21a and 21b may be taken simultaneously, but doing the homework and exam preparation for both courses will require considerable commitment of time and attention.

**Philosophy**: This course is greatly dependent upon your participation. Most of the mathematical concepts and techniques will be presented in class, with ample opportunity for questions and clarification. Please take advantage of these opportunities. The best lessons learned are those derived from discussion and practice.

Outside of class, it is essential that you do the assigned homework to learn the material. This will be the best preparation for interaction in the classroom and for the exams. Our course assistant will hold sections to clarify and expand upon material presented in class, especially the practical techniques and insights that help with the homework, and will allow a substantial fraction of time for your questions.

**Homework**: There will be a homework assignment given approximately every other lecture. All decisions regarding assignments turned in late will rest with our Teaching Assistant (Jeremy Marcq). Homework assignments and solutions will be posted on the course website as PDF documents which can be read using Adobe Acrobat. All submitted homework must be neat, stapled, and answers boxed where appropriate. You must provide evidence of your reasoning in your solutions of all homework problems. All assignments, due dates, and solutions will be posted in the **Course Calendar** on the course website.

Please note that the reading assigned with each homework is essential. Some topics not covered fully in class will be left to the reading and you will be expected to pick up those additional details.

**Academic Integrity**: You are responsible for understanding Harvard Summer School policies on academic integrity (http://www.summer.harvard.edu/policies/student-responsibilities) and how to use sources responsibly.

**Exams and grades**: There will be two 70-minute midterm exams, possibly scheduled outside of class time. These will occur during the third and fifth weeks of the course on dates to be determined. The (three hour) **Final Exam** is tentatively scheduled for **Friday, Aug 9 from 8:30am to 11:30am**. Practice exams and exam solutions will be posted on the course website. Your course grade will be computed according to:

.20(midterm exam 1) + .20(midterm exam 2) + .25(homework) + .35(final exam)

This scheme is subject to minor modification.

**Text**: ***Linear Algebra With Applications***, **4th Edition (2008)** or **3rd Edition (2005)** by Otto Bretscher, published by Prentice-Hall. The new **5th Edition (2013)** of the text may also be used, but HW problems are currently keyed to the 4th Edition. We will cover almost all topics in this book, and homework will be assigned from its large collection of exercises. The material is fundamentally the same in all editions and all homework assignments will be made available as printable PDFs. A key matching HW exercises in different editions is available on request. Additional supplements on various topics in differential equations will also be made available during the course.

**Calculators, computers**: Though not required for the course, you may find it very useful to have a matrix-capable calculator (such as a TI-83 Plus or higher) capable of finding the reduced row echelon form of a matrix (rref), matrix products, matrix inversion, and the calculation of determinants. Unless directed otherwise, calculators will be permitted on the exams.

**Accessibility**: The Summer School is committed to providing an accessible academic community. The Accessibility Office offers a variety of accommodations and services to students with documented disabilities. Please visit http://www.summer.harvard.edu/resources-policies/accessibility-services for more information.

## Mathematics S-21b syllabus

| Date | Text sections | Topics |
|---|---|---|
| Week 1 | 1.1: Introduction to Linear Systems<br>1.2: Matrices and Gauss-Jordan Elimination<br>1.3: On the Solutions of Linear Systems<br>2.1: Introduction to Linear Transformations and their Inverses<br>2.2: Linear Transformations in Geometry<br>2.3: The Inverse of a Linear Transformation<br>2.4: Matrix Products | Algebra and geometry of lines, planes; solving equations simultaneously; row reduction and row operations; rank of a matrix; homogeneous vs. inhomogeneous systems; Inverse of a matrix; linear transformations from $\mathbf{R}^m$ to $\mathbf{R}^n$; linearity; domain and codomain; invertibility; meaning of the columns of a matrix; rotations and dilations; shears; projections and reflections; matrix algebra, associativity and the composition of linear functions. |
| Week 2 | 3.1: Image and Kernel of a Linear Transformation<br>3.2: Subspaces of $\mathbf{R}^n$; Bases and Linear Independence<br>3.3: The Dimension of a Subspace of $\mathbf{R}^n$<br>3.4: Coordinates<br>4.1: Introduction to Linear Spaces<br>4.2: Linear Transformations and Isomorphisms<br>4.3: Coordinates in a Linear Space | Image and kernel of a linear transformation; linear combinations and the span of a set of vectors; subspaces; linear independence; basis; dimension of a subspace; bases for kernels and images; Rank-Nullity Theorem; coordinates of a vector relative to a basis; matrix of a linear transformation relative to a (nonstandard) basis; Examples of linear spaces other than $\mathbf{R}^n$, e.g. function spaces. Linear spaces; isomorphisms; coordinates; matrix of a general linear transformation relative to a basis. |
| Week 3 | **First hour exam**<br>5.1: Orthogonal Bases and Orthogonal Projections<br>5.2: Gram-Schmidt Process and QR Factorization<br>5.3: Orthogonal Transformations and Orthogonal Matrices<br>5.4: Least Squares and Data Fitting<br>5.5: Inner Product Spaces | Orthogonality (perpendicularity) of vectors in $\mathbf{R}^n$; length (norm) of a vector, unit vectors; orthogonal complements; orthogonal projections; orthonormal basis; angle between two vectors; Gram-Schmidt orthogonalization process; QR factorization; orthogonal transformation; orthogonal matrix; Least-squares approximation; normal equation. Inner products and norms in a general inner product space; Fourier analysis. |
| Week 4 | 6.1: Introduction to Determinants<br>6.2: Properties of the Determinant<br>6.3: Geometrical Interpretations of the Determinant; Cramer's Rule<br>7.1: Dynamical Systems and Eigenvectors: An Introductory Example<br>7.2: Finding the Eigenvalues of a Matrix<br>7.3: Finding the Eigenvectors of a Matrix<br>7.4: Diagonalization | Determinant of a (square) matrix; multilinearity; minors, cofactors, and adjoints; k-volumes; determinant as an expansion factor; Cramer's Rule; Discrete (linear) dynamical system; iteration of a matrix; eigenvectors and eigenvalues. Eigenvalues and eigenvectors of a (square) matrix; characteristic polynomial; algebraic and geometric multiplicities; similarity of matrices; Diagonalization and the existence of a basis of eigenvectors; powers of a matrix; eigenvalues of a linear transformation. |
| Week 5 | 7.5: Complex Eigenvalues<br>7.6: Stability<br>**Second hour exam**<br>8.1: Symmetric matrices<br>8.2: Quadratic Forms<br>9.1: An Introduction to Continuous Dynamical Systems | Complex numbers, polar form and related algebra; complex eigenvalues and eigenvectors; rotation-dilation matrices revisited; trace and determinant; stability of a discrete linear dynamical system; phase portraits. Spectral Theorem; symmetric matrices and diagonalization by an orthonormal basis; quadratic forms; positive definiteness of a matrix; principal axes; applications to ellipses and hyperbolas; 2nd derivative test for functions of several variables in terms of eigenvalues; Systems of linear differential equations and their solutions. |
| Week 6 | 9.2: The Complex Case: Euler’s Formula<br>9.3: Linear Differential Operators and Linear Differential Equations<br>10: Topics in Partial Differential Equations | Systems of linear differential equations and their solutions. Eigenfunctions, characteristic polynomials; kernel and image of a linear differential operator; solutions to homogeneous and inhomogeneous linear differential equations; Partial differential equations – Laplace’s equation, the heat equation, the wave equation. |
| Week 7 | Review and **Final Exam** | |