Math E-21c - Ordinary Differential Equations - Fall 2024

This course is a study of Ordinary Differential Equations (ODE's), including modeling physical systems. Topics include:

- Solution of First-order ODE's by Analytical, Graphical and Numerical Methods;
- Linear ODE's, Especially Second Order with Constant Coefficients;
- Basic ideas of Linear Algebra applied to ODEs - linear spaces, span, linear independence, basis for a subspace;
- Undetermined Coefficients and Variation of Parameters;
- Sinusoidal and Exponential Signals: Oscillations, Damping, Resonance;
- Complex Numbers and Exponentials;
- Fourier Series, Periodic Solutions;
- Delta Functions, Convolution, and Laplace Transform Methods;
- Matrix and First-order Linear Systems: Eigenvalues and Eigenvectors; and
- Non-linear Autonomous Systems: Critical Point Analysis and Phase Plane Diagrams.

Course Meeting Times:
Lectures (Robert Winters): Wednesdays, 8:10pm-10:10pm via Zoom (subject to change)

Optional Recitations (by Teaching Assistants Renée Chipman, Kris Lokere, Jeremy Marcq)

This course is being offered in an online web conference format (live or on demand). All lectures will be recorded and available via the Canvas site for the course.

Course website: http://math.rwinters.com/E21c and the related Canvas site (for accessing Lectures and recordings, submitting HW, and accessing HW and exam grades)

Prerequisites/Corequisites:
Single Variable Calculus is a prerequisite; *Multivariable Calculus* is recommended but not required; some familiarity with *Linear Algebra* is helpful but not required.

Texts: There is no specific required text for the course, but good references are:
(1) ***Elementary Differential Equations with Boundary Value Problems. 6th ed.*** by Edwards, C., and D. Penney. Upper Saddle River, NJ: Prentice Hall, 2008. ISBN: 9780136006138. [Note: The 5th Edition (ISBN: 9780131457744) or the 4th Edition will serve as well.]
(2) ***Differential Equations & Linear Algebra*** by Farlow, Hall, McDill, West. This text is published by Pearson and has ISBN #9780131860612.

We will provide our own Lecture Notes - which may be revised as the course proceeds. Additional supplements may also be provided.

Homework: Problem sets will be assigned each week and will be due the following week submitted online as a scanned PDF via Canvas. You are encouraged to discuss the homework with your fellow students, but you must write up the solutions by yourself without collaboration with others. (This is simply a matter of professional ethics.) Grading policies for the homework will be established after the first class meeting. Homework assignments and solutions will be posted on the course Calendar: **http://math.rwinters.com/E21c/calendar.htm**.

Solutions to the homework problems in PDF format will be made available via a password-protected web page linked from the Math E-21c course website. Selected problems may also be discussed in the problem sessions. Homework submitted after the posted deadline will be accepted only at the discretion of the Teaching Assistants.

Exams: There will be two midterm exams and a two-hour final examination. All exams will be conducted online within Canvas using *Proctorio*. Details will be posted on the course website.

Grading:
The final grade will be based on the following scheme (subject to minor modification):
25% homework, 40% midterm exams, 35% Final Exam

> **Note**: The "Graduate" credit option is primarily for students enrolled in certain Extension School graduate programs such as the "Math for Teaching" program. All other students (including high school students) should register for the "Undergraduate" option or the "Noncredit" option (if you will not be submitting homework or taking exams). Students registered for the Graduate credit option will be asked to submit additional work beyond the basic homework assignments.

The Harvard Extension School is committed to providing an accessible academic community. The Accessibility Office offers a variety of accommodations and services to students with documented disabilities. Please visit https://www.extension.harvard.edu/resources-policies/resources/disability-services-accessibility for more information.

Academic Integrity: You are responsible for understanding Harvard Extension School policies on academic integrity and how to use sources responsibly. Violations of academic integrity are taken very seriously. Review important information on academic integrity and student responsibilities here: https://extension.harvard.edu/for-students/student-policies-conduct/academic-integrity; for more on academic citation rules, visit Using Sources Effectively and Responsibly (https://extension.harvard.edu/for-students/support-and-services/using-sources-effectively-and-responsibly) and review the Harvard Guide to Using Sources (https://usingsources.fas.harvard.edu).

Publishing or Distributing Course Materials: Students may not post, publish, sell, or otherwise publicly distribute course materials without the written permission of the course instructor. Such materials include, but are not limited to, the following: lecture notes, lecture slides, video, or audio recordings, assignments, problem sets, examinations, other students' work, and answer keys. Students who sell, post, publish, or distribute course materials without written permission, whether for the purposes of soliciting answers or otherwise, may be subject to disciplinary action, up to and including requirement to withdraw. Further, students may not make video or audio recordings of class sessions for their own use without written permission of the instructor.

Proposed week-by-week syllabus (This may change somewhat as the course proceeds.)

Date	Topics
Sept 4	Basic notions, modeling with ordinary differential equations, separable equations, natural growth vs. logistic growth models, stability; 1st order linear ODEs, linear time-invariant ODEs, solution of 1st order linear ODEs using integrating factor.
Sept 11	Newton's Law of Cooling; homogeneous vs. inhomogeneous solutions, method of undetermined coefficients; Variation of Parameters; higher order linear ODEs; signal-response (or input-response) perspective.
Sept 18	Linear system response to exponential and sinusoidal input; gain, phase lag. Complex-valued equation associated to sinusoidal input. The algebra of complex numbers; the complex exponential; complex numbers, roots of unity. Applications to trigonometry, integration, and solving ODEs (complex replacement).
Sept 25	Basics of Linear Algebra: Subspaces, span, image and kernel, linear independence, basis, dimension, coordinates relative to a basis, function spaces and linear operators; 2nd order linear constant coefficient ODEs, characteristic polynomial, modes, independence of solutions, and superposition of solutions; Wronskian determinant; sinusoidal and exponential response; harmonic oscillator; complex characteristic roots.
Oct 2	Linear operators with constant coefficients (time invariant), exponential solutions, characteristic polynomial, distinct real roots, complex roots, repeated roots; Hooke's Law; complex replacement; Exponential Response Formula (ERF) and the Resonance Response Formula (RRF) for exponential and sinusoidal input signals. Gain and phase lag; resonance and forced harmonic motion; RLC circuits. **Midterm Exam I**
Oct 9	Exponential Response Formula (ERF), Resonance Response Formula (RRF); Resonance, Frequency response, LTI systems, superposition, RLC circuits; Exponential Shift Rule; Variation of Parameters for higher order systems; Discontinuous inputs.
Oct 16	Fourier series: orthogonality, inner products, orthogonal projection, Pythagorean Theorem; Applications to ODEs - harmonic response, resonance; Fourier series for periodic inputs; Fourier's Theorem and Fourier coefficients; square wave function; Sawtooth function; Differentiating and integrating Fourier series; Tips & Tricks: trig id, linear combination, shift
Oct 23	Generalized functions, generalized derivative, step and delta functions; Impulse and step responses; Laplace transform: basic properties, rules and sample calculations; t-domain vs s-domain; idea of how to solve ODEs by translating differential equations into algebraic equations; Step and delta functions. Impulse and step responses, generalized derivative; unit impulse response; time invariance;
Oct 30	Solution with initial conditions as $w*q$. Inverse transform; non-rest initial conditions for first order equations; worked examples of Laplace Transform and convolution
Nov 6	Linear algebra: linear independence, span, basis, coordinates; matrix of a linear transformation relative to a basis; Introduction to vector fields and systems of 1st order ODEs; reduction of order - nth order equations and systems of 1st order equations; matrix representation. First order linear systems of ODEs in matrix form, solution of uncoupled (diagonal) systems and evolution matrices; uncoupling a system (diagonalization) in case of real eigenvalues, evolution matrices; Sampler of Phase Plane diagrams for uncoupled, coupled, period, and stable sink linear systems and a nonlinear system.
Nov 13	Complex eigenvalues; Qualitative behavior of linear systems; phase plane; Solving System of 1st Order Linear Differential Equations, continued; repeated eigenvalues; decomposition of 1st order linear system into mode (block matrices); simple nonlinear system with shifted equilibrium. Matrix Methods for Solving Systems of 1st Order Linear Differential Equations; Phase portraits for the linear ODE examples. **Midterm Exam II**
Nov 20	Qualitative behavior of linear systems; phase plane [Eigenvalues vs coefficients; Complex eigenvalues; Repeated eigenvalues; Defective, complete; Trace-determinant plane; Stability]; simple nonlinear systems.
Dec 4	Nonlinear systems; linearization near equilibria, Jacobian matrices; nonlinear pendulum; autonomous systems, predator-prey systems.
Dec 11	Nonlinear systems; chaotic dynamical systems; Lorenz Attractor; Mathematical Theory of Epidemics
Dec 18	**Final Exam**