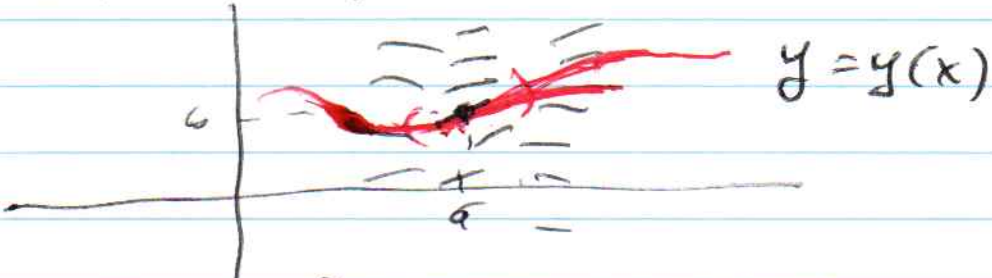
$$\frac{dy}{dx} = F(x,y)$$

[Lecture Notes #1, #2 used extensively during class.]



EXISTENCE + UNIQUENESS
OF SOLNS. to 1st order ODE's

Integrating factor [1st order Linear ODE's]

$$* \quad \frac{dx}{dt} + p(t)\,x = g(t) \qquad x(t)$$

$$\left[\frac{dy}{dx} + p(x)\,y = g(x) \right] \qquad y(x)$$

Integrating factor $v = v(t)$

$$v\frac{dx}{dt} + \boxed{vpx} = vg$$

Product Rule: $\frac{d}{dt}\left[vx\right] = v\frac{dx}{dt} + \boxed{x\frac{dv}{dt}}$

WANT: $\frac{dv}{dt} = vp$ Separable

$$\int \frac{dv}{v} = \int p\,dt \qquad \ln v = \int p(t)\,dt$$

$$v(t) = \boxed{e^{\int p(t)\,dt}}$$

$$\frac{d}{dt}\left[vx\right] = g(t)\, e^{\int p\,dt}$$

$t = a$ $\quad x(a)$ given $\qquad$ use "dummy" variable

$$v x \Big|_a^t = \int_{u=a}^{u=t} g(u) e^{\int p\,dt} \, du$$

$$\| $$

$$v(t)\, x(t) - v(a)\, x(a)$$

2nd Fund Thm of Calculus.

$$x(t) = \left[e^{-\int p\,dt}\right] \left[\int_a^t g(u) \left(e^{\int p\,dt}\right) du + v(a)\, x(a) \right]$$

Formula possible for soln, in terms of a definite integral. [see Revised Notes #1]

Input-Response Formulation [Linear ODE's]

$$\underbrace{\frac{d^n x}{dt^n} + p_{n-1}(t) \frac{d^{n-1}x}{dt^{n-1}} + \cdots + p_1(t) \frac{dx}{dt} + p_0(t)\, x}_{\text{System}} = \underbrace{g(t)}_{\text{Input}}$$

SOLUTION: $x(t)$ Response

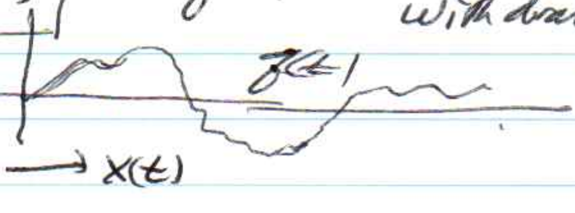
BANKING: $x = x(t)$ \$ in bank, Rate I

$$\frac{dx}{dt} = Ix \qquad x(0) = x_0 \quad \text{Initial Amount}$$

MAKE payment/WITHDRAWALS

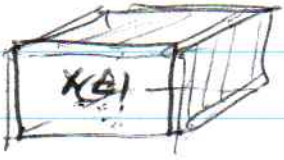
$$\boxed{\frac{dx}{dt} = Ix + g(t)} \qquad g(t) \text{ RATE of deposit/withdrawal}$$

$$\underbrace{\frac{dx}{dt} - Ix}_{\text{system}} = \underbrace{g(t)}_{\text{Input}}$$



DIFFUSION: Newton's Law of Cooling.

$y(t)$ outside temp.



$x(t)$ interior temp.

$$\boxed{\frac{dx}{dt} = K(y-x)}$$

$K > 0$

K coupling constant.

$$\frac{dx}{dt} = Ky - Kx$$

$$\boxed{\underbrace{\frac{dx}{dt} + Kx}_{\text{system}} = \underbrace{K\,y(t)}_{\text{Input.}}}$$

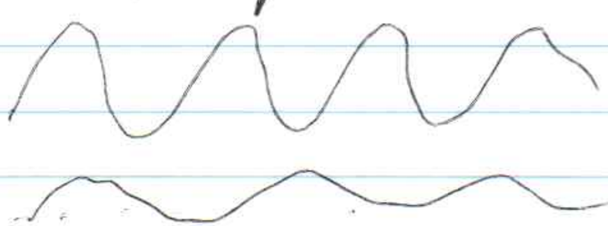
1st order, linear

If $y(t)$ sinusoidal

Expect $x(t)$ Respond sinusoidal.

Input.

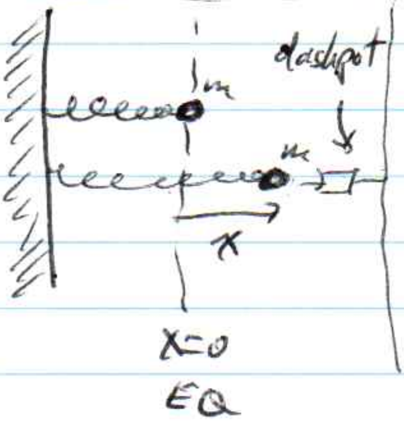
output



Expect Lag in response

RAPID VARIATION $\rightarrow$ Low Response Amplitude.

Hooke's Law + oscillation.



MASS-SPRING-DASHPOT w/(External) force

$F = -kx$ Hooke's Law.

ADD FRICTION

$F = -kx - cv$ w/ friction.

Apply external force

$$\boxed{F = -kx - cv + F_{ext}}$$

Newton $F = ma$ $\quad a = \frac{dv}{dt}$ $\quad v = \frac{dx}{dt}$

$$ma = -kx - cv + F_{ext} \qquad \dot{x} = \frac{dx}{dt}$$

$$m\frac{d^2x}{dt} = -kx - c\frac{dx}{dt} + F_{ext} \qquad \ddot{x} = \frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_{ext}$$

$$m\ddot{x} + c\dot{x} + kx = F_{ext}$$

$$\boxed{\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_{ext}}{m}}$$

$\underbrace{\text{2nd order ODE, LINEAR}}_{\text{System}}$, Inhomogeneous

Input

$x(t)$ Response

See Lecture Notes for additional details.