Math E-21b: Linear Algebra – Spring 2024

Course meetings: The class meets weekly on **Thursdays, 8:00pm to 10:00pm** [Harvard Hall 101 (in person) or via Zoom] or on-demand in Canvas. Optional problem sessions conducted by our Teaching Assistants will be scheduled after our first class meeting. Additional times for questions may follow as the course proceeds. An optional Q&A session with the instructor may also be scheduled at a day and time to be determined.

Instructor: **Robert Winters**, Lecturer of Mathematics, Harvard Extension School (formerly Harvard University, MIT, Brandeis University, and Wellesley College). Contact me at robert@math.rwinters.com.

Course website: **http://math.rwinters.com/E21b** (all assignments and solutions will be posted here) and **Canvas site**

Teaching Assistants: **Jeremy Marcq, Renée Chipman, Kris Lokere**

Prerequisites: Math E-16, or equivalent knowledge of algebra and calculus. You should be able to solve simple systems of equations and find the roots of polynomials. Also, you should be able to set up and solve simple differential equations. Math E-21a (or its equivalent) is not specifically necessary in order to take Math E-21b, but it will be very helpful if you have some familiarity with the algebra and geometry of lines and planes in $\mathbf{R}^2$, $\mathbf{R}^3$, and possibly $\mathbf{R}^n$, and the dot product of two vectors.

Note: The "Graduate" credit option is primarily for students enrolled in certain Extension School graduate programs such as the "Math for Teaching" program. All other students (including high school students) should register for the "Undergraduate" option or the Noncredit option (if you will not be submitting homework or taking exams). Students registered for "Graduate" credit will be asked to complete additional work on supplemental topics.

Philosophy: This course is greatly dependent upon your participation. Most of the mathematical concepts and techniques will be presented in class, with plenty of opportunity for questions and clarification, but the best lessons learned are those derived from discussion and practice. Outside of class, it is essential that you read the assigned text sections, do the assigned homework, and bring any questions to class or to the course assistant's section. Mathematics is not a spectator sport. Don't just crank through computations - think about the problems posed, your strategy, the meaning of the computations you perform, and the answers you get. This will help greatly in class and on the exams.

Homework: Problem sets will be posted each week on the course website (and later on Canvas) as a PDF file. Each assignment will be due online a few days after the following class submitted via the Canvas site as a single scanned PDF file that is clearly legible and of reasonable size. All policies regarding late homework will rest with the Teaching Assistants who will be reading and grading the assignments. All submitted homework must be neat, with answers boxed when appropriate. Solutions will be posted on the course website as PDF documents.

You are encouraged to discuss the homework with your fellow students, but you must write up the solutions by yourself without collaboration with others. (This is simply a matter of professional ethics.) Any unethical behavior on the homework (*such as copying solutions from a solutions manual*) may result in significant penalties or only exam scores being used in the calculation of your course grade – at the sole discretion of the instructor.

Please note that **the reading assigned with each homework is essential**. Some topics not covered fully in class will be left to the reading and you will be expected to pick up those additional details. Questions on the homework and the reading may also be directed to me at **Robert@math.rwinters.com**.

Exams and Grading: Two midterm exams will take place online via *Proctorio* during 24-hour windows approx. on Feb 29-Mar 1 and April 18-19. There will be a two-hour final exam online via *Proctorio* on May 9. Your course grade will be computed according to the following scheme, subject to minor modification: .25 (homework) + .40 (midterm exams) + .35 (final exam)

Text: ***Linear Algebra With Applications***, **4th Edition** (2008) by Otto Bretscher, published by Pearson/Prentice-Hall. A newer **5th Edition** (2012) is also acceptable, but assigned problems will be keyed to the 4th Edition. Older editions of the text may also be used. We will cover almost all topics in this book, and homework will be assigned from its large collection of exercises. The material is fundamentally the same in all editions and all homework assignments will be made available as printable PDFs. A key matching HW exercises in different editions is available on request. **Detailed Lecture Notes** and additional supplements on various topics will also be made available during the course.

Use of Technology: In some of the homework problems you will be asked not to use any technology (calculators or software packages). If no restriction is made, you may use the form of technology of your choice, e.g. TI calculators, Matlab, Maple, Mathematica. Make sure to have access to some form of technology. **Calculators (as opposed to computers) will be permitted on exams**, and it will be helpful if you are familiar with the matrix operations on a hand-held calculator, especially finding the reduced row-echelon form of a matrix. An effort will be made to write the exams in such a way that all problems may be solved without technology.

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Mathematics E-21b Topics

(This plan is ambitious and may have to be trimmed. Some topics may be omitted.)

Date (approx.)	**Text sections**	**Topics**
Thurs, Jan 25	1.1: Introduction to Linear Systems 1.2: Matrices, Vectors, and Gauss-Jordan Elimination 1.3: On the Solutions of Linear Systems; Matrix Algebra	Algebra and geometry of lines, planes; solving equations simultaneously; row reduction and row operations; rank of a matrix; homogeneous vs. inhomogeneous systems.
Thurs, Feb 1	2.1: Introduction to Linear Transformations and their Inverses 2.2: Linear Transformations in Geometry 2.3: Matrix Products	Linear transformations from $\mathbf{R}^m$ to $\mathbf{R}^n$; linearity; domain and codomain; invertibility; meaning of the columns of a matrix; rotations and dilations; shears; projections and reflections. Matrix algebra, associativity and the composition of linear functions.
Thurs, Feb 8	2.4: Matrix Products 3.1: Image and Kernel of a Linear Transformation 3.2: Subspaces of $\mathbf{R}^n$; Bases and Linear Independence	Inverse of a matrix; image and kernel of a linear transformation; linear combinations and the span of a set of vectors; subspaces; linear independence; basis.
Thurs, Feb 15	3.3: The Dimension of a Subspace of $\mathbf{R}^n$ 3.4: Coordinates relative to a basis	Dimension of a subspace; bases for kernels and images; Rank-nullity Theorem; coordinates of a vector relative to a basis; matrix of a linear transformation relative to a (nonstandard) basis.

Date (approx.)	**Text sections**	**Topics**
Thurs, Feb 22	4.1: Introduction to Linear Spaces 4.2: Linear Transformations and Isomorphisms 4.3: Coordinates in a Linear Space	Examples of linear spaces other than $\mathbf{R}^n$, e.g. function spaces. Linear spaces; isomorphisms; coordinates; matrix of a general linear transformation relative to a basis.
Thurs, Feb 29	5.1: Orthogonal Projections and Orthogonal Bases 5.2: Gram-Schmidt Process and QR Factorization **Midterm Exam 1**	Orthogonality (perpendicularity) of vectors in $\mathbf{R}^n$; length (norm) of a vector, unit vectors; orthogonal complements; orthogonal projections; orthonormal basis; angle between two vectors; Gram-Schmidt orthogonalization process; QR factorization.
Thurs, Mar 7	5.2: Gram-Schmidt Process and QR Factorization 5.3: Orthogonal Transformations and Orthogonal Matrices 5.4: Least Squares and Data Fitting	Gram-Schmidt orthogonalization process; QR factorization; orthogonal transformation; orthogonal matrix. Least-squares approximation; normal equation.
Thurs, Mar 21	6.1: Introduction to Determinants 6.2: Properties of the Determinant 6.3: Geometrical Interpretations of the Determinant; Cramer's Rule	Determinant of a (square) matrix; multilinearity; minors, cofactors, and adjoints; *k*-volumes; determinant as an expansion factor; Cramer's Rule.
Thurs, Mar 28	7.1: Dynamical Systems and Eigenvectors: An Introductory Example 7.2: Finding the Eigenvalues of a Matrix 7.3: Finding the Eigenvectors of a Matrix	Discrete (linear) dynamical system; iteration of a matrix; eigenvectors and eigenvalues of a (square) matrix; characteristic polynomial; algebraic and geometric multiplicities.
Thurs, Apr 4	7.4: Diagonalization 7.5: Complex Eigenvalues	Similarity of matrices; diagonalization and the existence of a basis of eigenvectors; powers of a matrix; eigenvalues of a linear transformation. Complex numbers; De Moivre's formula; rotation-dilation matrices revisited; trace and determinant.
Thurs, Apr 11	7.5: Complex Eigenvalues 7.6: Stability	Complex eigenvalues, repeated eigenvalues. Stability of a discrete linear dynamical system; phase portraits.
Thurs, Apr 18	8.1: Symmetric matrices 8.2: Quadratic Forms 8.3 Singular Values **Midterm Exam 2**	Spectral Theorem; symmetric matrices and diagonalization by an orthonormal basis; quadratic forms; positive definiteness of a matrix; principal axes; applications to ellipses and hyperbolas; 2nd derivative test for functions of several variables in terms of eigenvalues; Singular Values.
Thurs, Apr 25	9.1: An Introduction to Continuous Dynamical Systems 9.2: The Complex Case: Euler's Formula 9.3: Linear Differential Operators and Linear Differential Equations	Systems of linear differential equations and their solutions. Eigenfunctions, characteristic polynomials; kernel and image of a linear differential operator; solutions to homogeneous and inhomogeneous linear differential equations.
Thurs, May 2	9.3: Linear Differential Operators Nonlinear Systems of Differential Equations	Further applications in differential equations.
Thurs, May 11 – Fri, May 12	**FINAL EXAM**	-