Math E-21b: Linear Algebra – Spring 2017

Meeting Times: Class will meet in **Harvard Hall 201** every **Thursday from 7:40pm to 9:40pm** starting January 26. Weekly problem sessions will be scheduled based on the preferences of the class and the course assistants and will begin during the second week of the course. There will also be an optional Q&A session <u>before class each week</u> in Harvard Hall 103. Other times may be available on request for questions.

Instructor: **Robert Winters**, Lecturer of Mathematics, Mass. Institute of Technology (MIT), Concourse Program. Contact me at **Robert@math.rwinters.com**.

Course website: **http://math.rwinters.com/E21b** [All assignments and solutions will be posted here.]

Course Assistant(s): **Jeremy Marcq** & **Renée Chipman**

Prerequisites: Math E-16, or equivalent knowledge of algebra and calculus. You should be able to solve simple systems of equations and find the roots of polynomials. Also, you should be able to set up and solve simple differential equations. Math E-21a (or its equivalent) is not specifically necessary in order to take Math E-21b, but it will be very helpful if you have some familiarity with the algebra and geometry of lines and planes in $\mathbf{R}^2$, $\mathbf{R}^3$, and possibly $\mathbf{R}^n$, and the dot product of two vectors.

Philosophy: This course is greatly dependent upon your participation. Most of the mathematical concepts and techniques will be presented in class, with plenty of opportunity for questions and clarification, but the best lessons learned are those derived from discussion and practice. Outside of class, it is essential that you read the assigned text sections, do the assigned homework, and bring any questions to class or to the course assistant's section. <u>Mathematics is not a spectator sport</u>. Don't just crank through computations - think about the problems posed, your strategy, the meaning of the computations you perform, and the answers you get. This will help greatly in class and on the exams.

Homework: Problem sets will be posted each week in the Calendar section of the course website, including a PDF version for those using other texts. Each assignment will be due at the following class. Graded homework will be returned the class after that. Homework assignments should be turned in before class or immediately after class, but we will also have a mail slot on the 2nd floor of the Science Center where homework may be submitted if you cannot make it to class. All policies regarding late homework will rest with the course assistants who will be reading and grading the assignments. All submitted homework must be neat, with answers boxed when appropriate. Multiple pages <u>must</u> be stapled together. Solutions will be posted on the course website as PDF documents.

You are encouraged to discuss the homework with your fellow students, but <u>you must write up the solutions by yourself without collaboration with others</u>. (This is simply a matter of professional ethics.) Any unethical behavior on the homework (*such as copying solutions from a solutions manual*) may result in significant penalties or only exam scores being used in the calculation of your course grade.

Please note that **the reading assigned with each homework is essential**. Some topics not covered fully in class will be left to the reading and you will be expected to pick up those additional details. Questions on the homework and the reading may also be directed to me at **Robert@math.rwinters.com**.

Some of the homework problems will look different than problems discussed in class and in the text. This is not an accident. We want you to think about the material and learn to apply it in unfamiliar settings and interpret it in different ways. Only if you understand the material (as opposed to merely knowing it) will you be able to go beyond the information you are given.

Exams and Grading: Two in-class midterm exams are currently scheduled for the last half of class on <u>March 2</u> and <u>April 20</u>. There will be a two-hour final exam on <u>May 11</u>. Your course grade will be computed according to the following scheme, subject to minor modification: .20(hour exam 1) + .20(hour exam 2) + .20(homework) + .40(final exam)

Text: ***Linear Algebra With Applications***, **4th Edition** (2008) by Otto Bretscher, published by Pearson/Prentice-Hall. A newer **5th Edition** (2012) is also acceptable, but assigned problems will be keyed to the 4th Edition. Older editions of the text may also be used. We will cover almost all topics in this book, and homework will be assigned from its large collection of exercises. The material is fundamentally the same in all editions and all homework assignments will be made available as printable PDFs. A key matching HW exercises in different editions is available on request. Additional supplements on various topics will also be made available during the course.

Use of Technology: In some of the homework problems you will be asked not to use any technology (calculators or software packages). If no restriction is made, you may use the form of technology of your choice, e.g. TI calculators, Matlab, Maple, Mathematica. Make sure to have access to some form of technology. Calculators (as opposed to computers) will be permitted on exams, and it will be helpful if you are familiar with the matrix operations on a hand-held calculator, especially finding the reduced row-echelon form of a matrix. An effort will be made to write the exams in such a way that all problems may be solved without technology.

Mathematics E-21b Topics (This plan is ambitious and may have to be trimmed. Some topics may be omitted.)

Date (approx.)	**Text sections**	**Topics**
Thurs, Jan 26	1.1: Introduction to Linear Systems 1.2: Matrices, Vectors, and Gauss-Jordan Elimination 1.3: On the Solutions of Linear Systems; Matrix Algebra	Algebra and geometry of lines, planes; solving equations simultaneously; row reduction and row operations; rank of a matrix; homogeneous vs. inhomogeneous systems.
Thurs, Feb 2	2.1: Introduction to Linear Transformations and their Inverses 2.2: Linear Transformations in Geometry 2.3: Matrix Products	Linear transformations from $\mathbf{R}^m$ to $\mathbf{R}^n$; linearity; domain and codomain; invertibility; meaning of the columns of a matrix; rotations and dilations; shears; projections and reflections. Matrix algebra, associativity and the composition of linear functions.
Thurs, Feb 9	2.4: Matrix Products 3.1: Image and Kernel of a Linear Transformation 3.2: Subspaces of $\mathbf{R}^n$; Bases and Linear Independence	Inverse of a matrix; image and kernel of a linear transformation; linear combinations and the span of a set of vectors; subspaces; linear independence; basis.
Thurs, Feb 16	3.3: The Dimension of a Subspace of $\mathbf{R}^n$ 3.4: Coordinates	Dimension of a subspace; bases for kernels and images; Rank-nullity Theorem; coordinates of a vector relative to a basis; matrix of a linear transformation relative to a (nonstandard) basis.
Thurs, Feb 23	4.1: Introduction to Linear Spaces 4.2: Linear Transformations and Isomorphisms 4.3: Coordinates in a Linear Space	Examples of linear spaces other than $\mathbf{R}^n$, e.g. function spaces. Linear spaces; isomorphisms; coordinates; matrix of a general linear transformation relative to a basis.
Thurs, Mar 2	5.1: Orthogonal Projections and Orthogonal Bases 5.2: Gram-Schmidt Process and QR Factorization **Midterm Exam 1** (2^{nd} half of class)	Orthogonality (perpendicularity) of vectors in $\mathbf{R}^n$; length (norm) of a vector, unit vectors; orthogonal complements; orthogonal projections; orthonormal basis; angle between two vectors; Gram-Schmidt orthogonalization process; QR factorization.
Thurs, Mar 9	5.2: Gram-Schmidt Process and QR Factorization 5.3: Orthogonal Transformations and Orthogonal Matrices 5.4: Least Squares and Data Fitting	Gram-Schmidt orthogonalization process; QR factorization; orthogonal transformation; orthogonal matrix. Least-squares approximation; normal equation.
Thurs, Mar 23	6.1: Introduction to Determinants 6.2: Properties of the Determinant 6.3: Geometrical Interpretations of the Determinant; Cramer's Rule	Determinant of a (square) matrix; multilinearity; minors, cofactors, and adjoints; *k*-volumes; determinant as an expansion factor; Cramer's Rule.
Thurs, Mar 30	7.1: Dynamical Systems and Eigenvectors: An Introductory Example 7.2: Finding the Eigenvalues of a Matrix 7.3: Finding the Eigenvectors of a Matrix	Discrete (linear) dynamical system; iteration of a matrix; eigenvectors and eigenvalues of a (square) matrix; characteristic polynomial; algebraic and geometric multiplicities.
Thurs, Apr 6	7.4: Diagonalization 7.5: Complex Eigenvalues	Similarity of matrices; diagonalization and the existence of a basis of eigenvectors; powers of a matrix; eigenvalues of a linear transformation. Complex numbers; De Moivre's formula; rotation-dilation matrices revisited; trace and determinant.
Thurs, Apr 13	7.5: Complex Eigenvalues 7.6: Stability	Complex eigenvalues, repeated eigenvalues. Stability of a discrete linear dynamical system; phase portraits.
Thurs, Apr 20	8.1: Symmetric matrices 8.2: Quadratic Forms **Midterm Exam 2** (2^{nd} half of class)	Spectral Theorem; symmetric matrices and diagonalization by an orthonormal basis; quadratic forms; positive definiteness of a matrix; principal axes; applications to ellipses and hyperbolas; 2nd derivative test for functions of several variables in terms of eigenvalues.
Thurs, Apr 27	9.1: An Introduction to Continuous Dynamical Systems 9.2: The Complex Case: Euler's Formula 9.3: Linear Differential Operators and Linear Differential Equations	Systems of linear differential equations and their solutions. Eigenfunctions, characteristic polynomials; kernel and image of a linear differential operator; solutions to homogeneous and inhomogeneous linear differential equations.
Thurs, May 4	9.3: Linear Differential Operators Nonlinear Systems of Differential Equations	Further topics in differential equations.
Thurs, May 11	**FINAL EXAM**	-