

Math E-21b: Linear Algebra - Spring 2010

Meeting Times: Class will meet in Harvard Hall 201 every Thursday from 7:35pm to 9:35pm starting January 28. Weekly problem sessions will be scheduled as soon as course assistants are in place for the semester. The day(s) and time(s) of review sessions will be based on preferences of the class and the course assistants.

Instructor: Robert Winters, Wellesley College Mathematics Department. Contact me at Robert@math.rwinters.com.

Office Hours: There is expected to be a Q&A session before class each week at a nearby location. Other times may be available.

Course website: <http://math.rwinters.com/E21b> [All assignments and solutions will be posted here.]

Course Assistant(s): to be determined

Prerequisites: Math E-16, or equivalent knowledge of algebra and calculus. You should be able to solve simple systems of equations and find the roots of polynomials. Also, you should be able to set up and solve simple differential equations. Math E-21a (or its equivalent) is not specifically necessary in order to take Math E-21b, but it will be very helpful if you have some familiarity with the algebra and geometry of lines and planes in \mathbf{R}^2 , \mathbf{R}^3 , and possibly \mathbf{R}^n , and the dot product of two vectors.

Philosophy: This course is greatly dependent upon your participation. Most of the mathematical concepts and techniques will be presented in class, with plenty of opportunity for questions and clarification, but the best lessons learned are those derived from discussion and practice. Outside of class, it is essential that you read the assigned text sections, do the assigned homework, and bring any questions to class or to the course assistant's section. Mathematics is not a spectator sport. Don't just crank through computations - think about the problems posed, your strategy, the meaning of the computations you perform, and the answers you get. This will be the best preparation for interaction in the classroom and for the exams.

Homework: There will be weekly homework assignments that will be due the class after they are assigned. Homework assignments will be posted each week in the **Calendar** section of the course website, including a PDF version for those using other texts. Graded homework will be returned the class after that. Homework assignments should be turned in at the break during class, but we will also have a mail slot on the 3rd floor of the Science Center where homework may be submitted outside of class. Policies regarding homework will rest with the course assistants who will be reading and grading the assignments. Late homework will only be accepted with the prior consent of the course assistants. All homework must be neat, with answers boxed when appropriate. Multiple pages must be stapled together. Solutions will be posted on the course website as PDF documents.

Please note that the reading assigned with each homework is essential. Some topics not covered fully in class will be left to the reading and you will be expected to pick up those additional details. Questions on the homework and the reading may also be directed to me at Robert@math.rwinters.com.

Some of the homework problems will look different than problems discussed in class and in the text. This is not an accident. We want you to *think* about the material and learn to apply it in unfamiliar settings and interpret it in different ways. Only if you understand the material (as opposed to merely knowing it) will you be able to go beyond the information you are given.

Exams and Grading: There will be two in-class hour exams scheduled for March 4 and April 22. There will be a two-hour final exam on May 13. Your course grade will be computed according to the following scheme (subject to minor modification):
.20(hour exam 1) + .20(hour exam 2) + .25(homework) + .35(final exam)

Text: *Linear Algebra With Applications*, 3rd Edition, by Otto Bretscher, Prentice-Hall, 2005; ISBN 0-13-145334-3 for the text alone. It may also be purchased bundled with the student solutions manual. We will cover most of the topics in this book, though some topics will be omitted due to time constraints. Homework will be assigned from its large (and excellent) collection of exercises. Earlier editions of the text are acceptable since assignments will be posted as PDFs. There is also a new 4th Edition of the text available, but any edition will work well.

Use of Technology: In some of the homework problems you will be asked not to use any technology (calculators or software packages). If no restriction is made, you may use the form of technology of your choice, e.g. TI calculators, Matlab, Maple, Mathematica. Make sure to have access to some form of technology. Calculators (as opposed to computers) will be permitted on exams, and it will be helpful if you are familiar with the matrix operations on a hand-held calculator. An effort will be made to write the exams in such a way that all problems may be solved without technology.

Mathematics E-21b Topics (This plan is ambitious and may have to be trimmed. Some topics may be omitted.)

Date (approx.)	Text sections	Topics
Thurs, Jan 28	1.1: Introduction to Linear Systems 1.2: Matrices and Gauss-Jordan Elimination 1.3: On the Solutions of Linear Systems	Algebra and geometry of lines, planes; solving equations simultaneously; row reduction and row operations; rank of a matrix; homogeneous vs. inhomogeneous systems.
Thurs, Feb 4	2.1: Introduction to Linear Transformations and their Inverses 2.2: Linear Transformations in Geometry 2.3: The Inverse of a Linear Transformation	Inverse of a matrix; linear transformations from \mathbf{R}^m to \mathbf{R}^n ; linearity; domain and codomain; invertibility; meaning of the columns of a matrix; rotations and dilations; shears; projections and reflections.
Thurs, Feb 11	2.4: Matrix Products 3.1: Image and Kernel of a Linear Transformation 3.2: Subspaces of \mathbf{R}^n ; Bases and Linear Independence	Matrix algebra, associativity and the composition of linear functions; image and kernel of a linear transformation; linear combinations and the span of a set of vectors; subspaces; linear independence; basis.
Thurs, Feb 18	3.3: The Dimension of a Subspace of \mathbf{R}^n 3.4: Coordinates	Dimension of a subspace; bases for kernels and images; Rank-nullity Theorem; coordinates of a vector relative to a basis; matrix of a linear transformation relative to a (nonstandard) basis.
Thurs, Feb 25	4.1: Introduction to Linear Spaces 4.2: Linear Transformations and Isomorphisms 4.3: Coordinates in a Linear Space	Examples of linear spaces other than \mathbf{R}^n , e.g. function spaces. Linear spaces; isomorphisms; coordinates; matrix of a general linear transformation relative to a basis.
Thurs, Mar 4	5.1: Orthogonal Bases and Orthogonal Projections 5.2: Gram-Schmidt Process and QR Factorization Midterm Exam 1 (2 nd half of class)	Orthogonality (perpendicularity) of vectors in \mathbf{R}^n ; length (norm) of a vector, unit vectors; orthogonal complements; orthogonal projections; orthonormal basis; angle between two vectors; Gram-Schmidt orthogonalization process; QR factorization.
Thurs, Mar 11	5.2: Gram-Schmidt Process and QR Factorization 5.3: Orthogonal Transformations and Orthogonal Matrices 5.4: Least Squares and Data Fitting	Gram-Schmidt orthogonalization process; QR factorization; orthogonal transformation; orthogonal matrix. Least-squares approximation; normal equation.
Thurs, Mar 25	6.1: Introduction to Determinants 6.2: Properties of the Determinant 6.3: Geometrical Interpretations of the Determinant; Cramer's Rule	Determinant of a (square) matrix; multilinearity; minors, cofactors, and adjoints; k -volumes; determinant as an expansion factor; Cramer's Rule.
Thurs, Apr 1	7.1: Dynamical Systems and Eigenvectors: An Introductory Example 7.2: Finding the Eigenvalues of a Matrix 7.3: Finding the Eigenvectors of a Matrix	Discrete (linear) dynamical system; iteration of a matrix; eigenvectors and eigenvalues of a (square) matrix; characteristic polynomial; algebraic and geometric multiplicities.
Thurs, Apr 8	7.4: Diagonalization 7.5: Complex Eigenvalues	Similarity of matrices; diagonalization and the existence of a basis of eigenvectors; powers of a matrix; eigenvalues of a linear transformation. Complex numbers; De Moivre's formula; rotation-dilation matrices revisited; trace and determinant.
Thurs, Apr 15	7.5: Complex Eigenvalues, continued 7.6: Stability	Complex eigenvalues, repeated eigenvalues. Stability of a discrete linear dynamical system; phase portraits.
Thurs, Apr 22	8.1: Symmetric matrices 8.2: Quadratic Forms Midterm Exam 2 (2 nd half of class)	Spectral Theorem; symmetric matrices and diagonalization by an orthonormal basis; quadratic forms; positive definiteness of a matrix; principal axes; applications to ellipses and hyperbolas; 2 nd derivative test for functions of several variables in terms of eigenvalues.
Thurs, Apr 29	9.1: An Introduction to Continuous Dynamical Systems 9.2: The Complex Case: Euler's Formula 9.3: Linear Differential Operators	Systems of linear differential equations and their solutions. Eigenfunctions, characteristic polynomials; kernel and image of a linear differential operator; solutions to homogeneous and inhomogeneous linear differential equations.
Thurs, May 6	9.3: Linear Differential Operators Nonlinear systems of Differential Equations	Further topics in differential equations.
Thurs, May 13	FINAL EXAM	-