

## Mathematics E-21a — Multivariable Calculus — Fall 2025

**Instructor:** Robert Winters ([robert@math.rwinters.com](mailto:robert@math.rwinters.com)). We will also have 3 Teaching Assistants (**Renée Chipman, Kris Lokere, and Jeremy Marcq**) who will conduct the problem sessions and grade homework assignments.

**Recommended Prerequisites:** One full year of single variable calculus. Though it may be preferable to have seen some infinite series and differential equations, these topics are not necessary for successful completion of this course.

**About the Course:** We will cover at least the following topics: Vectors in  $\mathbf{R}^2$  and  $\mathbf{R}^3$  and vector methods, functions of several variables, differentiation and integration of functions of several variables, linear and quadratic approximation, optimization, parameterized curves and surfaces, vector fields, line and surface integrals, Green's Theorem, the Divergence Theorem and Stokes' Theorem.

**Note:** The "Graduate" credit option is primarily for students enrolled in certain Extension School graduate programs such as the "Math for Teaching" program. All other students (including high school students) should register for the "Undergraduate" option or the Noncredit option (if you will not be submitting homework or taking exams). Students registered for "Graduate" credit will be asked to complete additional work on supplemental topics.

**Textbook:** Multivariable Calculus: Concepts and Contexts by James Stewart - either the 4th Edition (published 2010 by Brooks-Cole, ISBN: 0495560545) OR the 3rd Edition (published 2005 by Brooks-Cole, ISBN: 0534410049) OR a comparable text. There are links on the course website to help you find an inexpensive copy.

**Note:** Each homework assignment will be posted as a PDF for those with other editions of the text.

**Course website:** <http://math.rwinters.com/E21a> and Canvas site

**Course meetings:** The class meets weekly on **Thursdays, 8:00pm to 10:00pm** [in person (Harvard Hall 101) or via Zoom] or on-demand in Canvas. Optional problem sessions conducted by our Teaching Assistants will be scheduled after our first class meeting. Additional times for questions may follow as the course proceeds. An optional Q&A session with the instructor may also be scheduled at a day and time to be determined.

**Homework:** Problem sets will be assigned each week and will be due online the following week (the Saturday after the following class). You are encouraged to discuss the homework with your fellow students, but you must write up the solutions by yourself without collaboration with others. (This is simply a matter of professional ethics.) Grading policies for the homework will be established after the first class meeting. Homework assignments and solutions will be posted on the course Calendar: <http://math.rwinters.com/E21a/calendar.htm>. Any unethical behavior on the homework (*such as the copying solutions from a solutions manual*) may result in only exam scores being used in the calculation of your course grade at the discretion of the instructor.

Solutions to the homework problems in PDF format will be made available via a password-protected web page linked from the Math E-21a course website. Selected problems may also be discussed in the problem sessions. Homework submitted after the day of the subsequent class meeting will be accepted only at the discretion of the teaching assistants.

**Exams:** There will be two 70 minute midterm exams and a two-hour Final Exam conducted online during a 24-hour window within the Canvas site using the Proctorio browser extension. The dates for the exams are:

**Midterm Exam 1:** Thurs, Oct 9 - Fri, Oct 10

**Midterm Exam 2:** Thurs, Nov 20 - Fri, Nov 21

**Final Exam:** Thurs, Dec 18

**Grading:** Your final grade will be based on your performance on the homework (20%), the two midterm exams (20% each), and the final exam (40%). These percentages are subject to minor modification.

**Computers and calculators:** The visualization of surfaces and other geometric objects is an important aspect of this course. Because computerized graphing programs can aid you in developing this ability, you are encouraged to employ them as part of the learning process. Use of mathematical software is nonetheless optional. Computers should be considered solely as an aid to the development of geometric intuition. Calculators will be permitted on the exams, but not computers. Various homework problems will ask you to sketch or otherwise describe various geometric objects. You are strongly advised to struggle with these first without electronic aids, as they may be quite trivial with a graphing program.

**Words of Caution and Advice:** This course may prove to be more demanding than your previous mathematics courses. The weekly assignments may be somewhat time-consuming, so you should plan now to set aside regular hours to wrestle with them. It is virtually impossible to do well in this course without working the homework assignments in a timely fashion. The course is somewhat fast-paced and new material builds on old, so do not fall behind. If you find yourself falling behind, please contact the instructor immediately so that options for assistance may be discussed.

When you are working your assignments, keep in mind that your success in this course will require more than just memorizing formulas and “plugging in values.” Numerical calculations are still important, but play a smaller role than in single-variable calculus. The key to success is understanding the underlying concepts and working enough problems so that you can employ them in any example thrown at you, especially homework and exam problems which differ significantly from material discussed in class.

The Harvard Extension School is committed to providing an accessible academic community. The Accessibility Office offers a variety of accommodations and services to students with documented disabilities. Please visit <https://www.extension.harvard.edu/resources-policies/resources/disability-services-accessibility> for more information.

**Academic Integrity:** You are responsible for understanding Harvard Extension School policies on academic integrity and how to use sources responsibly. Violations of academic integrity are taken very seriously. Review important information on academic integrity and student responsibilities here: <https://extension.harvard.edu/for-students/student-policies-conduct/academic-integrity>; for more on academic citation rules, visit Using Sources Effectively and Responsibly (<https://extension.harvard.edu/for-students/support-and-services/using-sources-effectively-and-responsibly>) and review the Harvard Guide to Using Sources (<https://usingsources.fas.harvard.edu>).

**Publishing or Distributing Course Materials:** Students may not post, publish, sell, or otherwise publicly distribute course materials without the written permission of the course instructor. Such materials include, but are not limited to, the following: lecture notes, lecture slides, video, or audio recordings, assignments, problem sets, examinations, other students’ work, and answer keys. Students who sell, post, publish, or distribute course materials without written permission, whether for the purposes of soliciting answers or otherwise, may be subject to disciplinary action, up to and including requirement to withdraw. Further, students may not make video or audio recordings of class sessions for their own use without written permission of the instructor.

**Detailed Syllabus (This may change as the course proceeds.)**

Date	Topics
Sept 4	Introduction to $\mathbf{R}^2$ and $\mathbf{R}^3$ . Points vs. vectors. Dot product in $\mathbf{R}^2$ , $\mathbf{R}^3$ , and $\mathbf{R}^n$ . Scalar and vector projections. Equation(s) of a line.
Sept 11	The cross product in $\mathbf{R}^3$ . Equations of lines and planes in $\mathbf{R}^3$ . Intersection of lines and planes. Functions, graphs, and surfaces. (Cylindrical and spherical coordinates.)
Sept 18	Vector-valued functions - parametrized curves in $\mathbf{R}^2$ and $\mathbf{R}^3$ . Velocity and acceleration vectors. Arclength. Equations of motion, tangential and normal components of acceleration.
Sept 25	Parametric surfaces. Functions of several variables; limits and continuity; graphs and level curves (contours) of a function of two variables. Partial derivatives.
Oct 2	Partial derivatives; differentiability; linear approximation and tangent planes; rate of change of a function along a parametrized curve; the Chain Rule.
Oct 9	Directional derivatives and the gradient vector. Level surfaces of a function of three variables. <b>Exam I</b>
Oct 16	Optimization; maximum and minimum values of a function. Constrained optimization and the Method of Lagrange Multipliers.
Oct 23	Integration over regions in $\mathbf{R}^2$ and $\mathbf{R}^3$ . Average value of a function. Iterated double integrals and the Fubini Theorem. Double integrals in polar coordinates.
Oct 30	Applications of double integrals: mass, moments, center of mass, moment of inertia, probability. Surface area. Triple integrals in Cartesian coordinates.
Nov 6	Triple integrals in cylindrical coordinates and spherical coordinates; applications of triple integrals. Change of variables in multiple integrals; Jacobian matrices.
Nov 13	Vector fields in $\mathbf{R}^2$ and $\mathbf{R}^3$ . Line integrals and work done by a variable force along a parametrized curve; Fundamental Theorem of Line Integrals, independence of path, conservative vector fields.
Nov 20	Green’s Theorem. <b>Exam II</b>
Dec 4	Curl and divergence of a vector field; surface integrals; flux of a vector field through a surface.
Dec 11	Stokes’ Theorem; the Divergence Theorem
Dec 18	<b>Final Exam</b>