# Mathematics E-21a — Multivariable Calculus — Fall 2014

**Instructor**: **Robert Winters <robert@math.rwinters.com>**. We will also have two Course Assistants, **Renée Chipman and Jeremy Marcq**, who will conduct the weekly problem sessions and grade homework assignments.

**Recommended Prerequisites**: One full year of single variable calculus. Though it may be preferable to have seen some infinite series and differential equations, these topics are not necessary for successful completion of this course.

**About the Course**: We will cover at least the following topics: Vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$ and vector methods, functions of several variables, differentiation and integration of functions of several variables, linear and quadratic approximation, optimization, parameterized curves and surfaces, vector fields, line and surface integrals, Green's Theorem, the Divergence Theorem and Stokes' Theorem.

**Textbook**: <u>Multivariable Calculus: Concepts and Contexts</u> by James Stewart - either the <u>4th Edition</u> (published 2010 by Brooks-Cole, ISBN: 0495560545) OR the <u>3rd Edition</u> (published 2005 by Brooks-Cole, ISBN: 0534410049). The text should be available at The Coop or via the Internet.
**Note**: If you can find an inexpensive copy of the 2nd Edition of the Stewart text or a comparable text, this should also work fine. Each homework assignment will be posted as a PDF for those with other editions of the text.

**Course website**: **http://math.rwinters.com/E21a**

**Course meetings**: The class meets weekly on **Thursdays, 7:40pm to 9:40pm, in Harvard Hall 202**.

Optional weekly problem sessions conducted by our course assistants will be scheduled after our first class meeting. Additional times for questions may follow as the course proceeds. An optional **Q&A session before class** with the instructor will take place each week (room to be determined) after the first week.

**Homework**: Problem sets will be assigned each week and will be due the following week in class. You are encouraged to discuss the homework with your fellow students, but you must write up the solutions by yourself <u>without collaboration with others</u>. (This is simply a matter of professional ethics.) Grading policies for the homework will be established after the first class meeting. Homework assignments and solutions will be posted on the course Calendar: **http://math.rwinters.com/E21a/calendar.htm**. Any unethical behavior on the homework (*such as the copying solutions from a solutions manual*) may result in only exam scores being used in the calculation of your course grade.

Solutions to the homework problems in PDF format will be made available via a password-protected web page linked from the Math E-21a course website. Selected problems may also be discussed in the problem sessions. Homework submitted after the day of the subsequent class meeting will be accepted only at the discretion of the course assistants.

**Exams**: There will be two one-hour midterms and a (longer) final exam for the course. The tentative dates for the midterm exams are:

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| **Midterm Exam 1:** <u>**Thursday, October 9**</u> | **Midterm Exam 2:** <u>**Thursday, November 20**</u> |

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**Final Exam:** <u>**Thursday, December 18**</u>

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**Grading**: Your final grade will be based on your performance on the homework (20%), the two midterm exams (20% each), and the final exam (40%). These percentages are subject to minor modification.

**Computers and calculators**: The visualization of surfaces and other geometric objects is an important aspect of this course. Because computerized graphing programs can aid you in developing this ability, you are encouraged to employ them as part of the learning process. Use of mathematical software is nonetheless optional. Computers should be considered solely as an aid to the development of geometric intuition. <u>Calculators will be permitted on the exams, but not computers</u>. Various homework problems will ask you to sketch or otherwise describe various geometric objects. You are strongly advised to struggle with these first without electronic aids, as they may be quite trivial with a graphing program.

**Words of Caution and Advice**: This course may prove to be more demanding than your previous mathematics courses. The weekly assignments may be somewhat time-consuming, so you should plan now to set aside regular hours to wrestle with them. It is virtually impossible to do well in this course without working the homework assignments in a timely fashion. The course is somewhat fast-paced and new material builds on old, so do not fall behind. If you find yourself falling behind, please contact the instructor immediately so that options for assistance may be discussed.

When you are working your assignments, keep in mind that your success in this course will require more than just memorizing formulas and "plugging in values." Numerical calculations are still important, but play a smaller role than in single-variable calculus. The key to success is understanding the underlying concepts and working enough problems so that you can employ them in any example thrown at you, especially homework and exam problems which differ significantly from material discussed in class.

### Detailed Syllabus (This may change as the course proceeds.)

| Date | Topics | Text sections |
|------|--------|---------------|
| Sept 4 | Introduction to $\mathbf{R}^2$ and $\mathbf{R}^3$. Points vs. vectors. Dot product in $\mathbf{R}^2$, $\mathbf{R}^3$, and $\mathbf{R}^n$. Scalar and vector projections. Equation(s) of a line. | 9.1 - 9.3 |
| Sept 11 | The cross product in $\mathbf{R}^3$. Equations of lines and planes in $\mathbf{R}^3$. Intersection of lines and planes. Functions, graphs, and surfaces. (Cylindrical and spherical coordinates.) | 9.4 – 9.6; (9.7) |
| Sept 18 | Vector-valued functions - parametrized curves in $\mathbf{R}^2$ and $\mathbf{R}^3$. Velocity and acceleration vectors. Arclength. Equations of motion, tangential and normal components of acceleration. | 10.1 – 10.4 |
| Sept 25 | Parametric surfaces. Functions of several variables; limits and continuity; graphs and level curves (contours) of a function of two variables. Partial derivatives. | 10.5; 11.1 – 11.3 |
| Oct 2 | Partial derivatives; differentiability; linear approximation and tangent planes; rate of change of a function along a parametrized curve; the Chain Rule. | 11.3 – 11.4 |
| Oct 9 | Directional derivatives and the gradient vector. Level surfaces of a function of three variables.   **Midterm Exam I** | 11.5 – 11.6 |
| Oct 16 | Optimization; maximum and minimum values of a function. Constrained optimization and the Method of Lagrange Multipliers. | 11.7 – 11.8 |
| Oct 23 | Integration over regions in $\mathbf{R}^2$ and $\mathbf{R}^3$. Average value of a function. Iterated double integrals and the Fubini Theorem. Double integrals in polar coordinates. | 12.1 – 12.4 |
| Oct 30 | Applications of double integrals: mass, moments, center of mass, moment of inertia, probability. Surface area. Triple integrals in Cartesian coordinates. | 12.5 – 12.7 |
| Nov 6 | Triple integrals in cylindrical coordinates and spherical coordinates; applications of triple integrals. Change of variables in multiple integrals; Jacobian matrices. | 12.8 – 12.9 |
| Nov 13 | Vector fields in $\mathbf{R}^2$ and $\mathbf{R}^3$. Line integrals and work done by a variable force along a parametrized curve; Fundamental Theorem of Line Integrals, independence of path, conservative vector fields. | 13.1 – 13.2 |
| Nov 20 | Green's Theorem.   **Midterm Exam II** | 13.3 – 13.4 |
| Dec 4 | Curl and divergence of a vector field; surface integrals; flux of a vector field through a surface. | 13.5 – 13.6 |
| Dec 11 | Stokes' Theorem; the Divergence Theorem | 13.7 – 13.8 |
| Dec 18 | **Final Exam** | |