Math E-21a – Fall 2011 – HW #9 problems

Problems keyed to the 4th Edition

Problems to turn in on Thurs, Nov 3:

Section 12.2:

12. Calculate the iterated integral: $\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dx\,dy$

20. Calculate the double integral: $\iint_R \frac{x}{1+xy}\,dA, \quad R=[0,1]\times[0,1]=\{(x,y):0\le x\le 1, 0\le y\le 1\}$

Section 12.3:

8. Evaluate the double integral: $\iint_D \frac{y}{x^5+1}\,dA, \quad D=\{(x,y)\,|\,0\le x\le 1, 0\le y\le x^2\}$

28. Find the volume of the solid bounded by the planes $z=x$, $y=x$, $x+y=2$, and $z=0$.

30. Find the volume of the solid bounded by the cylinder $y^2+z^2=4$ and the planes $x=2y$, $x=0$, $z=0$ in the first octant.

32. Find the volume of the solid bounded by the cylinders $x^2+y^2=r^2$ and $y^2+z^2=r^2$.

In problems 48 and 50, evaluate the integral by reversing the order of integration. [Sketch the domain first!!]

48. $\int_0^{\sqrt{\pi}}\int_y^{\sqrt{\pi}}\cos(x^2)\,dx\,dy$ 50. $\int_0^1\int_x^1 e^{x/y}\,dy\,dx$

Section 12.4:

10. Evaluate the given integral by changing to polar coordinates: $\iint_R\sqrt{4-x^2-y^2}\,dA$, where $R=\{(x,y)\,|\,x^2+y^2\le 4, x\ge 0\}$.

In problems 18 and 20, use polar coordinates to find the volume of the given solid.

18. Inside the sphere $x^2+y^2+z^2=16$ and outside the cylinder $x^2+y^2=4$.

20. Bounded by the paraboloid $z=1+2x^2+2y^2$ and the plane $z=7$ in the first octant.

24. (a) A cylindrical drill with radius r_1 is used to bore a hole through the center of a sphere with radius r_2. Find the volume of the ring-shaped solid that remains.
    (b) Express the volume in part (a) in terms of the height h of the ring.
    [Notice that the volume depends only on h, not on r_1 or r_2.]

26. Use a double integral to find the area of the region inside the cardioid $r=1+\cos\theta$ and outside the circle $r=3\cos\theta$. [Note: You will definitely want to draw these curves correctly before setting up any integrals.]

36. (a) We define the improper integral (over the entire plane $\mathbf{R}^2$)

$$I=\iint_{\mathbf{R}^2} e^{-(x^2+y^2)}dA=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dy\,dx=\lim_{a\to\infty}\iint_{D_a} e^{-(x^2+y^2)}dA$$

where D_a is the disk with radius a and center the origin. Show that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dy\,dx=\pi$.

(b) An equivalent definition of the improper integral in part (a) is $I=\iint_{\mathbf{R}^2} e^{-(x^2+y^2)}dA=\lim_{a\to\infty}\iint_{S_a} e^{-(x^2+y^2)}dA$

where S_a is the square with vertices $(\pm a,\pm a)$. Use this to show that $\int_{-\infty}^{\infty} e^{-x^2}dx\int_{-\infty}^{\infty} e^{-y^2}dy=\pi$.

(c) Deduce that $\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi}$.

(d) By making the change of variable $t=\sqrt{2}\,x$, show that $\int_{-\infty}^{\infty} e^{-x^2/2}dx=\sqrt{2\pi}$. [This is a fundamental result for probability and statistics, specifically the normal probability distribution.]

Section 12.5:

2. Electric charge is distributed over the disk $x^2 + y^2 \le 4$ so that the charge density at (x, y) is $\sigma(x, y) = x + y + x^2 + y^2$ (measured in coulombs per square meter). Find the total charge on the disk.

6. Find the mass and center of mass of the lamina that occupies the triangular region *D* bounded by the lines $x = 0$, $y = x$, and $2x + y = 6$ with density function $\rho(x, y) = x^2$. [See text for definition of center of mass.]

12. A lamina occupies the part of the disk $x^2 + y^2 \le 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to the square of the distance from the origin. [See text or class notes for definition of center of mass.]

For additional practice:

Section 12.2:

3. Calculate the iterated integral: $\int_1^3 \int_0^1 (1 + 4xy)\, dx\, dy$

9. Calculate the iterated integral: $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x} \right) dx\, dy$

35. Find the average value of *f* over the given rectangle:
$f(x, y) = x^2 y$, R has vertices $(-1,0), (-1,5), (1,5), (1,0)$.

Section 12.3:

4. Evaluate the iterated integral: $\int_0^1 \int_x^{2-x} (x^2 - y)\, dy\, dx$

6. Evaluate the iterated integral: $\int_0^1 \int_0^v \sqrt{1 - v^2}\, du\, dv$

In problems 7 and 21, evaluate the double integral.

7. $\iint_D y^2\, dA$, $D = \{(x, y) \mid -1 \le y \le 1, -y - 2 \le x \le y\}$

21. $\iint_D (2x - y)\, dA$, D is bounded by the circle with center the origin and radius 2.

In problems 24, 25, and 31, find the volume of the given solid.

24. Under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

25. Under the surface $z = xy$ and above the triangle with vertices $(1,1), (4,1)$, and $(1,2)$.

31. Bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y = z$, $x = 0$, $z = 0$ in the first octant.

47. Evaluate the integral by reversing the order of integration: $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

Section 12.4:

9. Evaluate the given integral by changing to polar coordinates: $\iint_R \cos(x^2+y^2)\,dA$, where R is the region that lies above the x-axis within the circle $x^2+y^2=9$.

In problems 19 and 21, use polar coordinates to find the volume of the given solid.

19. A sphere of radius a.

21. Above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$.

25. Use a double integral to find the area of the region: One loop of the rose $r=\cos 3\theta$.

29. Evaluate the iterated integral $\int_0^1\int_y^{\sqrt{2-y^2}}(x+y)\,dx\,dy$

Section 12.5:

1. Electric charge is distributed over the rectangle $1\le x\le 3, 0\le y\le 2$ so that the charge density at (x,y) is $\sigma(x,y)=2xy+y^2$ (measured in coulombs per square meter). Find the total charge on the rectangle.

5. Find the mass and center of mass of the lamina that occupies the triangular region D with vertices $(0,0)$, $(2,1)$, $(0,3)$ with density function $\rho(x,y)=x+y$.

11. A lamina occupies the part of the disk $x^2+y^2\le 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis.