

Math E-21a – Fall 2011 – HW #3 problems

Problems keyed to the 4<sup>th</sup> Edition

Problems to turn in on Thurs, Sept 22:

Section 9.6:

11. Sketch the graph of the function  $f(x, y) = 6 - 3x - 2y$ .
12. Sketch the graph of the function  $f(x, y) = \cos x$ .
34. Find an equation for the surface consisting of all points  $P$  for which the distance from  $P$  to the  $x$ -axis is twice the distance from  $P$  to the  $yz$ -plane. Identify the surface.

Chapter 9 Review Exercises:

8. Show that if  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  are in  $\mathbf{R}^3$  (denoted by  $V_3$  in the text) then  $(\mathbf{a} \times \mathbf{b}) \cdot [(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})] = [\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})]^2$ .
10. Given the points  $A(1, 0, 1)$ ,  $B(2, 3, 0)$ ,  $C(-1, 1, 4)$ , and  $D(0, 3, 2)$ , find the volume of the parallelepiped with adjacent edges  $AB$ ,  $AC$ , and  $AD$ .
25. (a) Find the distance between the planes  $3x + y - 4z = 2$  and  $3x + y - 4z = 24$ .  
(b) Find the distance from the origin to the line  $x = 1 + t$ ,  $y = 2 - t$ ,  $z = -1 + 2t$ .
26. (a) Find an equation of the plane that passes through the points  $A(2, 1, 1)$ ,  $B(-1, -1, 10)$ , and  $C(1, 3, -4)$ .  
(b) Find symmetric equations for the line through  $B$  that is perpendicular to the plane in part (a).  
(c) A second plane passes through  $(2, 0, 4)$  and has normal vector  $\langle 2, -4, -3 \rangle$ . Show that the acute angle between the planes is approximately  $43^\circ$ .  
(d) Find parametric equations for the line of intersection of the two planes.
36. Identify and sketch the graph of the surface  $y^2 + z^2 = 1 + x^2$ . Include several traces (cross-sections) in your sketch.

Section 10.1:

In problems 6-9, sketch the curve with the given vector equation. Indicate with an arrow the direction in which  $t$  increases.

6.  $\mathbf{r}(t) = \langle t^3, t^2 \rangle$       7.  $\mathbf{r}(t) = \langle t, 2 - t, 2t \rangle$       8.  $\mathbf{r}(t) = \langle \sin(\pi t), t, \cos(\pi t) \rangle$       9.  $\mathbf{r}(t) = \langle 1, \cos t, 2 \sin t \rangle$

44. Two particles travel along the space curves  $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$      $\mathbf{r}_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle$ .

Do the particles collide? Do their paths intersect?

**Section 10.2:** [This may require reading of the text in addition to what was covered in lecture. In particular, the velocity vector for a vector-valued (position) vector is found by differentiating each of the component functions.]

In problems 4 and 6,

- a) Sketch the plane curve with the given vector equation.
- b) Find  $\mathbf{r}'(t)$ .
- c) Sketch the position vector  $\mathbf{r}(t)$  and the tangent vector  $\mathbf{r}'(t)$  for the given value of  $t$ .

4.  $\mathbf{r}(t) = \langle 1 + t, \sqrt{t} \rangle$ ,  $t = 1$       6.  $\mathbf{r}(t) = e^t \mathbf{i} + e^{-t} \mathbf{j}$ ,  $t = 0$

24. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point:  $x = \ln t$ ,  $y = 2\sqrt{t}$ ,  $z = t^2$ ;  $(0, 2, 1)$

32. At what point do the curves  $\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle$  and  $\mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle$  intersect?

Find their angle of intersection correct to the nearest degree.

**Additional practice problems:**

**Section 9.6:**

13. Sketch the graph of the function  $f(x, y) = y^2 + 1$ .

15. Match the function with its graph (labeled I-VI). Give reasons for your choices.

a)  $f(x, y) = |x| + |y|$

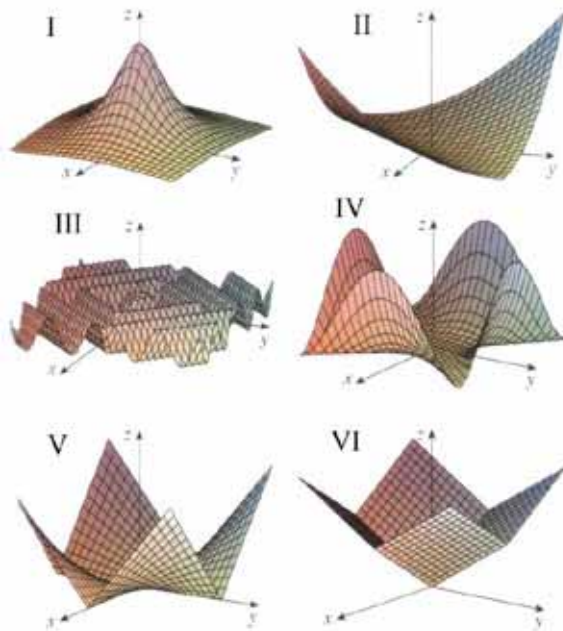
b)  $f(x, y) = |xy|$

c)  $f(x, y) = \frac{1}{1+x^2+y^2}$

d)  $f(x, y) = (x^2 - y^2)^2$

e)  $f(x, y) = (x - y)^2$

f)  $f(x, y) = \sin(|x| + |y|)$



23. a) What does the equation  $x^2 + y^2 = 1$  represent as a curve in  $\mathbf{R}^2$ ?

b) What does it represent as a surface in  $\mathbf{R}^3$ ?

c) What does the equation  $x^2 + z^2 = 1$  represent?

**Chapter 9 True False Quiz Questions:**

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$  (text uses  $V_3$ ),  $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ .
2. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$ ,  $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$ .
3. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$ ,  $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{v} \times \mathbf{u}\|$ .
4. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$  and any scalar  $k$ ,  $k(\mathbf{u} \cdot \mathbf{v}) = (k\mathbf{u}) \cdot \mathbf{v}$ .
5. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$  and any scalar  $k$ ,  $k(\mathbf{u} \times \mathbf{v}) = (k\mathbf{u}) \times \mathbf{v}$ .
6. For any vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  in  $\mathbf{R}^3$ ,  $(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$ .
7. For any vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  in  $\mathbf{R}^3$ ,  $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ .
8. For any vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  in  $\mathbf{R}^3$ ,  $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ .
9. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$ ,  $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$ .
10. For any vectors  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbf{R}^3$ ,  $(\mathbf{u} + \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$ .
11. The cross product of two unit vectors is a unit vector.
12. A linear equation  $Ax + By + Cz = D$  represents a line in space.
13. The set of points  $\{(x, y, z) \mid x^2 + y^2 = 1\}$  is a circle.
14. If  $\mathbf{u} = \langle u_1, u_2 \rangle$  and  $\mathbf{v} = \langle v_1, v_2 \rangle$ , then  $\mathbf{u} \cdot \mathbf{v} = \langle u_1 v_1, u_2 v_2 \rangle$ .
15. If  $\mathbf{u} \cdot \mathbf{v} = 0$ , then  $\mathbf{u} = \mathbf{0}$  or  $\mathbf{v} = \mathbf{0}$ .
16. If  $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ , then  $\mathbf{u} = \mathbf{0}$  or  $\mathbf{v} = \mathbf{0}$ .

17. If  $\mathbf{u} \cdot \mathbf{v} = 0$  and  $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ , then  $\mathbf{u} = \mathbf{0}$  or  $\mathbf{v} = \mathbf{0}$ .

18. If  $\mathbf{u}$  and  $\mathbf{v}$ , in  $\mathbf{R}^3$ ,  $|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \|\mathbf{v}\|$ .

**Section 10.1:**

In problems 19-24, match the parametric equations with the graphs (labeled I-VI). Give reasons for your choices:

19.  $x = t \cos t, \quad y = t, \quad z = t \sin t$

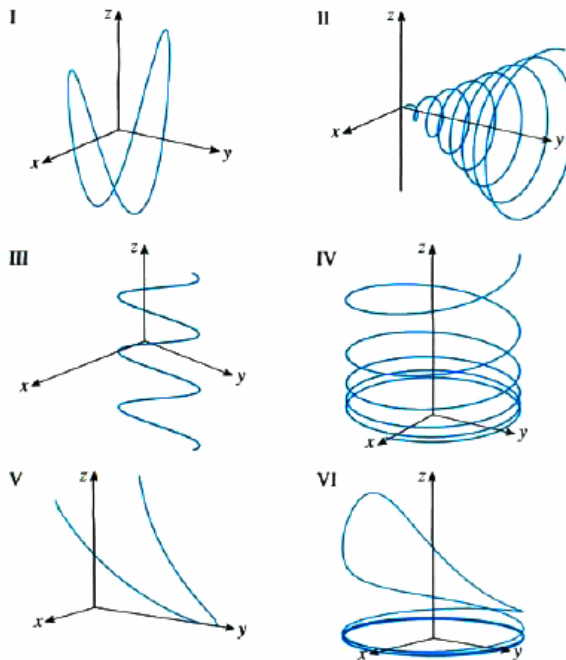
20.  $x = \cos t, \quad y = \sin t, \quad z = \frac{1}{1+t^2}$

21.  $x = t, \quad y = \frac{1}{1+t^2}, \quad z = t^2$

22.  $x = \cos t, \quad y = \sin t, \quad z = \cos 2t$

23.  $x = \cos 8t, \quad y = \sin 8t, \quad z = e^{0.8t}, \quad t \geq 0$

24.  $x = \cos^2 t, \quad y = \sin^2 t, \quad z = t$



25. Show that the curve with parametric equations  $x = t \cos t, y = t \sin t, z = t$  lies on the cone  $z^2 = x^2 + y^2$ , and use this fact to help sketch the curve.

27. At what point does the curve  $\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$  intersect the paraboloid  $z = x^2 + y^2$ ?

37. Find a vector function that represents the curve of intersection of the two surfaces:

The cone  $z = \sqrt{x^2 + y^2}$  and the plane  $z = 1 + y$ .

**Section 10.2:**

In problems 5 and 7,

a) Sketch the plane curve with the given vector equation.

b) Find  $\mathbf{r}'(t)$ .

c) Sketch the position vector  $\mathbf{r}(t)$  and the tangent vector  $\mathbf{r}'(t)$  for the given value of  $t$ .

5.  $\mathbf{r}(t) = \sin t \mathbf{i} + 2 \cos t \mathbf{j}, \quad t = \pi/4$

7.  $\mathbf{r}(t) = e^t \mathbf{i} + e^{3t} \mathbf{j}, \quad t = 0$