

Math E-21a – Fall 2009 – HW #2 problems

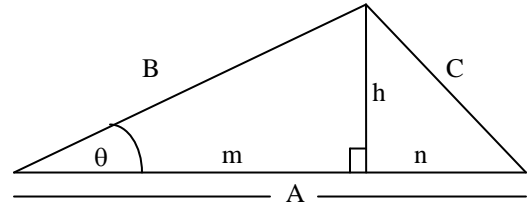
Problems to turn in on Thurs, Sept 17:

Basic Concept Problems:

(a) Prove the Pythagorean Theorem. [*Hint*: This can be done, for example, by considering the areas of plane figures like squares and triangles.]

(b) Prove the Law of Cosines using only the Pythagorean Theorem and some algebra and trigonometry. You may find the diagram shown at the right useful.

[Show that $C^2 = A^2 + B^2 - 2AB \cos \theta$]



Section 9.3:

16. Find, correct to the nearest degree, the three angles of the triangle with the vertices $D(0, 1, 1)$, $E(-2, 4, 3)$, and $F(1, 2, -1)$.

24. Find the scalar and vector projections of \mathbf{b} onto \mathbf{a} : $\mathbf{a} = \langle 1, 2 \rangle$, $\mathbf{b} = \langle -4, 1 \rangle$

25. Find the scalar and vector projections of \mathbf{b} onto \mathbf{a} : $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$

37. Find the angle between the diagonal of a cube and one of its edges.

40. If $\mathbf{c} = \|\mathbf{a}\|\mathbf{b} + \|\mathbf{b}\|\mathbf{a}$, where \mathbf{a} , \mathbf{b} , and \mathbf{c} are all nonzero vectors, show that \mathbf{c} bisects the angle between \mathbf{a} and \mathbf{b} .

Section 9.4:

14. Find two unit vectors orthogonal to both $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + \mathbf{k}$.

16. Find the area of the parallelogram with vertices $K(1, 2, 3)$, $L(1, 3, 6)$, $M(3, 8, 6)$, and $N(3, 7, 3)$.

18. a) Find a vector orthogonal to the plane through the three points P , Q , and R , and

b) find the area of the triangle PQR with the points $P(2, 1, 5)$, $Q(-1, 3, 4)$, and $R(3, 0, 6)$.

27. a) Let P be a point not on the line L that passes through the points Q and R . Show that the distance d from the point P to the line L is

$$d = \frac{\|\mathbf{a} \times \mathbf{b}\|}{\|\mathbf{a}\|}$$

where $\mathbf{a} = \overline{QR}$ and $\mathbf{b} = \overline{QP}$.

b) Use the formula in part (a) to find the distance from the point $P(1, 1, 1)$ to the line through $Q(0, 6, 8)$ and $R(-1, 4, 7)$.

Section 9.5:

10. Find parametric equations and symmetric equations for the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

18. Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

$$L_1: x = 1 + 2t, \quad y = 3t, \quad z = 2 - t \qquad L_2: x = -1 + s, \quad y = 4 + s, \quad z = 1 + 3s$$

30. Find an equation of the plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$.

48. Find the distance from the point to the given plane: $(3, -2, 7)$, $4x - 6y + z = 5$

50. Find the distance between the given parallel planes: $3x + 6y - 9z = 4$, $x + 2y - 3z = 1$

Additional practice problems (not to be turned in):

Section 9.3:

17. Determine whether the given vectors are orthogonal, parallel, or neither.

a) $\mathbf{a} = \langle -5, 3, 7 \rangle$, $\mathbf{b} = \langle 6, -8, 2 \rangle$

b) $\mathbf{a} = \langle 4, 6 \rangle$, $\mathbf{b} = \langle -3, 2 \rangle$

c) $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$

d) $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = -3\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$

20. For what values of b are the vectors $\langle -6, b, 2 \rangle$ and $\langle b, b^2, b \rangle$ orthogonal?

35. Use a scalar projection to show that the distance from a point $P_1(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Use this formula to find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$.

Section 9.4:

In problems 7 and 11, find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both \mathbf{a} and \mathbf{b} .

7. $\mathbf{a} = \langle 1, 2, 0 \rangle$, $\mathbf{b} = \langle 0, 3, 1 \rangle$

11. $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

21. Find the volume of the parallelepiped determined by the vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} .

$\mathbf{a} = \langle 6, 3, -1 \rangle$, $\mathbf{b} = \langle 0, 1, 2 \rangle$, $\mathbf{c} = \langle 4, -2, 5 \rangle$

30. Prove the following formula for the vector triple product:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

Section 9.5:

In problems 3 and 5, find a vector equation and parametric equations for the line:

3. The line through the point $(-2, 4, 10)$ and parallel to the vector $\langle 3, 1, -8 \rangle$.

5. The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$.

In problems 21, 25, and 27, find an equation of the plane:

21. The plane through the point $(6, 3, 2)$ and perpendicular to the vector $\langle -2, 1, 5 \rangle$.

25. The plane through the points $(0, 1, 1)$, $(1, 0, 1)$, and $(1, 1, 0)$.

27. The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$