Math E-21a – Fall 2011 – HW #14 problems

Do these problems, but <u>don't turn them in</u>:

Section 13.7:

In problem 10, use Stokes' Theorem to evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$. The curve C is oriented counterclockwise as viewed from above, i.e. from the positive z-axis.

10. $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + 2z\,\mathbf{j} + 3y\,\mathbf{k}$, C is the curve of intersection of the plane $x+z=5$ and the cylinder $x^2+y^2=9$.

In problem 14, verify that Stokes' Theorem is true for the given vector field **F** and surface S.

14. $\mathbf{F}(x,y,z) = -2yz\,\mathbf{i} + y\,\mathbf{j} + 3x\,\mathbf{k}$, S is the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane $z=1$, oriented upward.

Section 13.8:

In problems 2 and 4, verify that the Divergence Theorem is true for the vector field **F** on the region E.

2. $\mathbf{F}(x,y,z) = x^2\,\mathbf{i} + xy\,\mathbf{j} + z\,\mathbf{k}$, E is the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

4. $\mathbf{F}(x,y,z) = \left\langle x^2, -y, z \right\rangle$, E is the solid cylinder $y^2+z^2 \le 9$, $0 \le x \le 2$.

In problems 6, 8, and 10, use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$; that is, calculate the flux of **F** across S.

6. $\mathbf{F}(x,y,z) = x^2yz\,\mathbf{i} + xy^2z\,\mathbf{j} + xyz^2\,\mathbf{k}$, S is the surface of the box enclosed by the planes $x=0$, $x=a$, $y=0$, $y=b$, $z=0$, $y=c$ where a, b, and c are positive numbers.

8. $\mathbf{F}(x,y,z) = (x^3+y^3)\,\mathbf{i} + (y^3+z^3)\,\mathbf{j} + (z^3+x^3)\,\mathbf{k}$, S is the sphere with center the origin and radius 2.

10. $\mathbf{F}(x,y,z) = x^2y\,\mathbf{i} + xy^2\,\mathbf{j} + 2xyz\,\mathbf{k}$, S is the surface of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$, and $x+2y+z=2$.

18. Let $\mathbf{F}(x,y,z) = z\tan^{-1}(y^2)\,\mathbf{i} + z^3\ln(x^2+1)\,\mathbf{j} + z\,\mathbf{k}$. Find the flux of **F** across the part of the paraboloid $x^2+y^2+z=2$ that lies above the plane $z=1$ and is oriented upward.

For additional practice:

Section 13.7:

In problems 7 and 9, use Stokes' Theorem to evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$. In each case, C is oriented counterclockwise as viewed from above, i.e. from the positive z-axis.

7. $\mathbf{F}(x,y,z)=(x+y^2)\,\mathbf{i}+(y+z^2)\,\mathbf{j}+(z+x^2)\,\mathbf{k}$, C is the triangle with vertices (1,0,0), (0,1,0), and (0,0,1).

9. $\mathbf{F}(x,y,z)=yz\,\mathbf{i}+2xz\,\mathbf{j}+e^{xy}\,\mathbf{k}$, C is the circle $x^2+y^2=16,\ z=5$.

In problems 13 and 15, verify that Stokes' Theorem is true for the given vector field **F** and surface S.

13. $\mathbf{F}(x,y,z)=-y\,\mathbf{i}+x\,\mathbf{j}-2\,\mathbf{k}$, S is the cone $z^2=x^2+y^2,\quad 0\le z\le 4$, oriented downward.

15. $\mathbf{F}(x,y,z)=y\,\mathbf{i}+z\,\mathbf{j}+x\,\mathbf{k}$, S is the hemisphere $x^2+y^2+z^2=1,\quad y\ge 0$, oriented in the direction of the positive y-axis.

Section 13.8:

In problems 1 and 3, verify that the Divergence Theorem is true for the vector field **F** on the region E.

1. $\mathbf{F}(x,y,z)=3x\,\mathbf{i}+xy\,\mathbf{j}+2xz\,\mathbf{k}$, E is the cube bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, and $z=1$.

3. $\mathbf{F}(x,y,z)=\langle z,y,x\rangle$, E is the solid ball $x^2+y^2+z^2\le 16$.

In problems 7 and 9, use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$; that is, calculate the flux of **F** across S.

7. $\mathbf{F}(x,y,z)=3xy^2\,\mathbf{i}+xe^z\,\mathbf{j}+z^3\,\mathbf{k}$, S is the surface of the solid bounded by the cylinder $y^2+z^2=1$ and the planes $x=-1$ and $x=2$.

9. $\mathbf{F}(x,y,z)=x^2\sin y\,\mathbf{i}+x\cos y\,\mathbf{j}-xz\sin y\,\mathbf{k}$, S is the "fat sphere" $x^8+y^8+z^8=8$.

17. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z)=z^2x\,\mathbf{i}+(\tfrac{1}{3}y^3+\tan z)\,\mathbf{j}+(x^2z+y^2)\,\mathbf{k}$ and S is the top half of the sphere $x^2+y^2+z^2=1$.
[*Hint*: Note that S is not a closed surface. First compute integrals over S_1 and S_2, where S_1 is the disk $x^2+y^2\le 1$ in the xy-plane, oriented downward, and $S_2=S\cup S_1$.]