Math E-21a – Fall 2018 – HW #14 problems

Do these problems, but <u>don't turn them in</u>. These topics will likely appear on the Final Exam. Solutions will be posted, but do the problems before consulting the posted solutions.

Section 13.7:

In problem 10, use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$. The curve C is oriented counterclockwise as viewed from above, i.e. from the positive z-axis.

10. $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + 2z\,\mathbf{j} + 3y\,\mathbf{k}$, C is the curve of intersection of the plane $x + z = 5$ and the cylinder $x^2 + y^2 = 9$.

In problem 14, verify that Stokes' Theorem is true for the given vector field **F** and surface *S*.

14. $\mathbf{F}(x, y, z) = -2yz\,\mathbf{i} + y\,\mathbf{j} + 3x\,\mathbf{k}$, S is the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane $z = 1$, oriented upward.

Section 13.8:

In problems 2 and 4, verify that the Divergence Theorem is true for the vector field **F** on the region *E*.

2. $\mathbf{F}(x, y, z) = x^2\,\mathbf{i} + xy\,\mathbf{j} + z\,\mathbf{k}$, E is the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

4. $\mathbf{F}(x, y, z) = \langle x^2, -y, z \rangle$, E is the solid cylinder $y^2 + z^2 \le 9$, $0 \le x \le 2$.

In problems 6, 8, and 10, use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of **F** across *S*.

6. $\mathbf{F}(x, y, z) = x^2yz\,\mathbf{i} + xy^2z\,\mathbf{j} + xyz^2\,\mathbf{k}$, S is the surface of the box enclosed by the planes $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$, $y = c$ where *a*, *b*, and *c* are positive numbers.

8. $\mathbf{F}(x, y, z) = (x^3 + y^3)\,\mathbf{i} + (y^3 + z^3)\,\mathbf{j} + (z^3 + x^3)\,\mathbf{k}$, S is the sphere with center the origin and radius 2.

10. $\mathbf{F}(x, y, z) = x^2y\,\mathbf{i} + xy^2\,\mathbf{j} + 2xyz\,\mathbf{k}$, S is the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + z = 2$.

18. Let $\mathbf{F}(x, y, z) = z\tan^{-1}(y^2)\,\mathbf{i} + z^3\ln(x^2 + 1)\,\mathbf{j} + z\,\mathbf{k}$. Find the flux of **F** across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and is oriented upward.

For additional practice:

Section 13.7:

In problems 7 and 9, use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$. In each case, C is oriented counterclockwise as viewed from above, i.e. from the positive z-axis.

7. $\mathbf{F}(x, y, z) = (x + y^2)\,\mathbf{i} + (y + z^2)\,\mathbf{j} + (z + x^2)\,\mathbf{k}$, C is the triangle with vertices (1,0,0), (0,1,0), and (0,0,1).

9. $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + 2xz\,\mathbf{j} + e^{xy}\,\mathbf{k}$, C is the circle $x^2 + y^2 = 16,\ z = 5$.

In problems 13 and 15, verify that Stokes' Theorem is true for the given vector field **F** and surface S.

13. $\mathbf{F}(x, y, z) = -y\,\mathbf{i} + x\,\mathbf{j} - 2\,\mathbf{k}$, S is the cone $z^2 = x^2 + y^2$, $0 \le z \le 4$, oriented downward.

15. $\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} + x\,\mathbf{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 1$, $y \ge 0$, oriented in the direction of the positive y-axis.

Section 13.8:

In problems 1 and 3, verify that the Divergence Theorem is true for the vector field **F** on the region E.

1. $\mathbf{F}(x, y, z) = 3x\,\mathbf{i} + xy\,\mathbf{j} + 2xz\,\mathbf{k}$, E is the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

3. $\mathbf{F}(x, y, z) = \langle z, y, x \rangle$, E is the solid ball $x^2 + y^2 + z^2 \le 16$.

In problems 7 and 9, use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of **F** across S.

7. $\mathbf{F}(x, y, z) = 3xy^2\,\mathbf{i} + xe^z\,\mathbf{j} + z^3\,\mathbf{k}$, S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

9. $\mathbf{F}(x, y, z) = x^2 \sin y\,\mathbf{i} + x \cos y\,\mathbf{j} - xz \sin y\,\mathbf{k}$, S is the "fat sphere" $x^8 + y^8 + z^8 = 8$.

17. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = z^2 x\,\mathbf{i} + (\frac{1}{3} y^3 + \tan z)\,\mathbf{j} + (x^2 z + y^2)\,\mathbf{k}$ and S is the top half of the sphere $x^2 + y^2 + z^2 = 1$.
[*Hint*: Note that S is not a closed surface. First compute integrals over S_1 and S_2, where S_1 is the disk $x^2 + y^2 \le 1$ in the xy-plane, oriented downward, and $S_2 = S \cup S_1$.]