Math E-21a – Fall 2018 – HW #13 problems

To be turned in Thurs, Dec 13 (or online no later than Dec 15):

Section 12.6:

2. Find the area of the part of the plane $2x-5y+z=10$ that lies above the triangle with vertices $(0,0)$, $(0,6)$, and $(4,0)$.

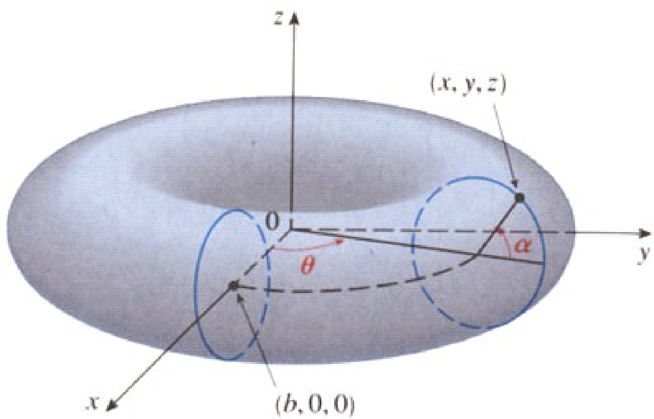
4. Find the area of the part of the plane with vector equation $\mathbf{r}(u,v)=\langle u+v,\,2-3u,\,1+u-v\rangle$ that is given by $0\le u\le 2,\,-1\le v\le 1$.

6. Find the area of the part of the surface $z=1+3x+2y^2$ that lies above the triangle with vertices (0,0), (0,1), and (2,1).

22. The figure shows the torus obtained by rotating about the z-axis the circle in the xz-plane with center $(b,0,0)$ and radius $a<b$.
Parametric equations for the torus are

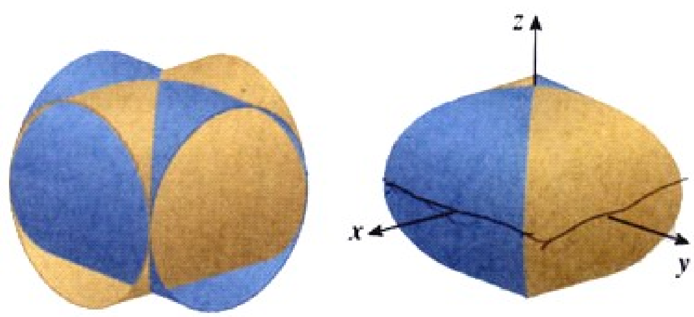
$$\begin{Bmatrix} x=b\cos\theta+a\cos\alpha\cos\theta \\ y=b\sin\theta+a\cos\alpha\sin\theta \\ z=a\sin\alpha \end{Bmatrix}$$

where θ and α are the angles shown in the figure. Find the surface area of the torus.



26. The figure shows the surface created when the cylinder $y^2+z^2=1$ intersects the cylinder $x^2+z^2=1$. Find the area of this surface.

[Hint: Find the surface area of one of the eight identical pieces of this surface, e.g. the part above the triangular region bounded by the lines $y=x$, $y=-x$, and $x=1$.]



Section 13.6:

10. Evaluate the surface integral $\iint_S xy\,dS$ where S is the triangular region with vertices $(1,0,0)$, $(0,2,0)$, and $(0,0,2)$. [*Note*: The surface S is a portion of a plane.]

16. Evaluate the surface integral $\iint_S y^2\,dS$, where S is the part of sphere $x^2+y^2+z^2=4$ that lies inside the cylinder $x^2+y^2=1$ and above the xy-plane.

24. Evaluate the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x,y,z)=x\,\mathbf{i}+y\,\mathbf{j}+z^4\,\mathbf{k}$ where S is the part of the cone $z=\sqrt{x^2+y^2}$ beneath the plane $z=1$ with downward orientation. [*Note*: $d\mathbf{S}=\mathbf{n}\,dS$ where $\mathbf{n}$ denotes the unit normal vector to the surface consistent with the orientation of the surface.]

26. Evaluate the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x,y,z) = xz\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$ where S is the hemisphere $x^2+y^2+z^2=25, \quad y \ge 0$, oriented in the direction of the positive y-axis.

30. Evaluate the (flux) surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x,y,z) = y\,\mathbf{i} + (z-y)\,\mathbf{j} + x\,\mathbf{k}$ where S is the surface of the tetrahedron with vertices (0,0,0), (1,0,0), (0,1,0), and (0,0,1) with outward orientation.

38. Find the mass of a thin funnel in the shape of a cone $z=\sqrt{x^2+y^2}$, $1 \le z \le 4$, if its density function is $\sigma(x,y,z) = 10 - z$.

For additional practice:
Section 12.6:
3. Find the area of the part of the plane $3x+2y+z=6$ that lies in the first octant.

10. Find the area of the part of the paraboloid $x = y^2+z^2$ that lies inside the cylinder $y^2+z^2=9$.
[<u>*Hint*</u>: Try modifying the method for a graph $z = f(x,y)$ for this case where we have $x = f(y,z)$.]

11. Find the area of the part of the sphere $x^2+y^2+z^2=b^2$ that lies inside the cylinder $x^2+y^2=a^2$, where $0<a<b$.

23. Use Definition 4 and the parametric equations for a surface of revolution (see Equations 10.5.3) to derive Formula 7.
<u>*Note 1*</u>: Definition 4 states that if a smooth parametric surface S is given by
$\mathbf{r}(u,v) = x(u,v)\,\mathbf{i} + y(u,v)\,\mathbf{j} + z(u,v)\,\mathbf{k} \quad (u,v)\in D$, and S is covered just once as (u,v) ranges throughout the parameter domain D, then the surface area of S is:

$$A(S) = \iint_D \|\mathbf{r}_u \times \mathbf{r}_v\|\,dA \text{ where } \mathbf{r}_u = \frac{\partial \mathbf{r}}{\partial u} = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\right) \text{ and } \mathbf{r}_v = \frac{\partial \mathbf{r}}{\partial v} = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\right).$$

<u>*Note 2*</u>: The (easily derived) parametric equations (10.5.3) for a surface of revolution are: $\left\{\begin{matrix} x = x \\ y = f(x)\cos\theta \\ z = f(x)\sin\theta \end{matrix}\right\}$

where $\{y = f(x), a \le x \le b\}$ and $0 \le \theta \le 2\pi$.

<u>*Note 3*</u>: Formula 7 gives the area of a surface of revolution: $A = 2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$.

27. Find the area of the part of the sphere $x^2+y^2+z^2=4z$ that lies inside the paraboloid $z=x^2+y^2$.

Section 13.6:
9. Evaluate the surface integral $\iint_S x^2yz\,dS$ where S is the part of the plane $z=1+2x+3y$ that lies above the rectangle $[0,3]\times[0,2]$.

11. Evaluate the surface integral $\iint_S yz\,dS$ where S is the part of the plane $x+y+z=1$ that lies in the first octant.

13. Evaluate the surface integral $\iint_S x^2 z^2 \, dS$ where S is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$.

14. Evaluate the surface integral $\iint_S z \, dS$ where S is the surface $x = y + 2z^2, \quad 0 \le y \le 1, \quad 0 \le z \le 1$.

15. Evaluate the surface integral $\iint_S y \, dS$ where S is the part of the paraboloid $y = x^2 + z^2$ that lies inside the cylinder $x^2 + z^2 = 4$.

17. Evaluate the surface integral $\iint_S (x^2 z + y^2 z) \, dS$ where S is the hemisphere $x^2 + y^2 + z^2 = 4, \quad z \ge 0$.

18. Evaluate the surface integral $\iint_S xz \, dS$ where S is the boundary of the region enclosed by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$ and $x + y = 5$.

21. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + yz\,\mathbf{j} + zx\,\mathbf{k}$ where S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \le x \le 1, 0 \le y \le 1$ and has upward orientation.

23. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = xze^y\,\mathbf{i} - xze^y\,\mathbf{j} + z\,\mathbf{k}$ where S is the part of the plane $x + y + z = 1$ in the first octant and has downward orientation.

27. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = y\,\mathbf{j} - z\,\mathbf{k}$, where S consists of the paraboloid $y = x^2 + z^2, 0 \le y \le 1$, and the disk $x^2 + z^2 \le 1, y = 1$.

37. Find the center of mass of the hemisphere $x^2 + y^2 + z^2 = a^2, z \ge 0$, if it has constant density.

40. Lets S be the part of the sphere $x^2 + y^2 + z^2 = 25$ that lies above the plane $z = 4$. If S has constant density k, find (a) the center of mass and (b) the moment of inertia about the z-axis. [See text for definitions of center of mass and moment of inertia in this context.]

46. The temperature at a point in a ball with conductivity K is inversely proportional to the distance from the center of the ball. Find the rate of heat flow across a sphere S of radius a with center at the center of the ball.