Math E-21a – Fall 2018 – HW #11 problems

Problems due in class on Thurs, Nov 29 (or online no later than Sat, Dec 1):

Section 12.9:

16. Evaluate the integral $\iint_R (4x+8y)\,dA$, where R is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1)$, and $(1,5)$. Use the coordinate transformation $\{x=\frac{1}{4}(u+v),\ y=\frac{1}{4}(v-3u)\}$.

17. Evaluate the integral $\iint_R x^2\,dA$, bounded by the ellipse $9x^2+4y^2=36$. Use the coordinate transformation $\{x=2u,\ y=3v\}$.

[**Note**: In both of these exercises, though the transformation is provided, you should be able to come up with the appropriate coordinate transformation by considering the geometry of the region involved.]

Section 13.1:

6. Sketch the vector field $\mathbf{F}(x,y)=\dfrac{y\,\mathbf{i}-x\,\mathbf{j}}{\sqrt{x^2+y^2}}$.

25. Find the gradient vector field ∇f of $f(x,y)=x^2-y$ and sketch it.

There's a good java-based tool for showing vector fields and flows in $\mathbf{R}^2$ at http://math.rice.edu/~dfield/dfpp.html. Choose the PPLANE option. You can enter new *x* and *y* component functions for the vector field or change the size of the window. To see a trajectory (flow), just click on a point in the phase-plane.

Section 13.2:

In problems 2, 7, and 10, evaluate the line integral, where *C* is the given curve.

2. $\int_C xy\,ds \quad C: x=t^2, y=2t,\ 0\le t\le 1$.

7. $\int_C xy\,dx+(x-y)\,dy$, *C* consists of line segments from $(0,0)$ to $(2,0)$, and from $(2,0)$ to $(3,2)$.

10. $\int_C xyz^2\,ds$, *C* is the line segment from $(-1,5,0)$ to $(1,6,4)$.

20. Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z)=(x+y)\,\mathbf{i}+(y-z)\,\mathbf{j}+z^2\,\mathbf{k}$ and where the curve *C* is given by the vector function $\mathbf{r}(t)=t^2\,\mathbf{i}+t^3\,\mathbf{j}+t^2\,\mathbf{k},\ 0\le t\le 1$.

28. Use a graph of the vector field **F** and the curve *C* to guess whether the line integral of **F** over *C* is positive, negative, or zero. Then evaluate the line integral.

$$\mathbf{F}(x,y)=\frac{x}{\sqrt{x^2+y^2}}\mathbf{i}+\frac{y}{\sqrt{x^2+y^2}}\mathbf{j},\quad C \text{ is the parabola } y=1+x^2 \text{ from } (-1, 2) \text{ to } (1, 2).$$

34. A thin wire has the shape of the first-quadrant part of the circle with center the origin and radius *a*. If the density function is $\sigma(x,y)=kxy$, find the mass and center of mass of the wire. [Here *k* is a constant.]

Section 13.3:

8. Determine whether or not the vector field $\mathbf{F}(x,y)=(3x^2-2y^2)\,\mathbf{i}+(4xy+3)\,\mathbf{j}$ is a conservative vector field. If it is, find a function *f* such that $\mathbf{F}=\nabla f$.

10. Determine whether or not the vector field $\mathbf{F}(x,y)=(xy\cos xy+\sin xy)\,\mathbf{i}+(x^2\cos xy)\,\mathbf{j}$ is a conservative vector field. If it is, find a function *f* such that $\mathbf{F}=\nabla f$.

In problems 12 and 16, find a function *f* such that $\mathbf{F}=\nabla f$ and use this function to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ along the given curve *C*.

12. $\mathbf{F}(x,y)=x^2\,\mathbf{i}+y^2\,\mathbf{j}$, *C* is the arc of the parabola $y=2x^2$ from $(-1,2)$ to $(2,8)$.

16. $\mathbf{F}(x,y,z)=(2xz+y^2)\,\mathbf{i}+2xy\,\mathbf{j}+(x^2+3z^2)\,\mathbf{k},\quad C: x=t^2, y=t+1, z=2t-1,\quad 0\le t\le 1$.

20. Show that the line integral is independent of path and evaluate the integral.

$$\int_C (1-ye^{-x})\,dx+e^{-x}\,dy,\quad C \text{ is any path from } (0,1) \text{ to } (1,2).$$

For additional practice:
Section 12.9:

19. Evaluate the integral $\iint_R xy\,dA$, where R is the region in the first quadrant bounded by the lines $y = x$ and $y = 3x$ and the hyperbolas $xy = 1$ and $xy = 3$. Use the coordinate transformation $\{x = \frac{u}{v},\ y = v\}$.

21. a) Evaluate $\iiint_E dV$, where E is the solid enclosed by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
Use the coordinate transformation $\{x = au,\ y = bv,\ z = cw\}$.
b) Earth is not a perfect sphere; rotation has resulted in flattening at the poles. So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use part (a) to estimate the volume of Earth.

Section 13.1:

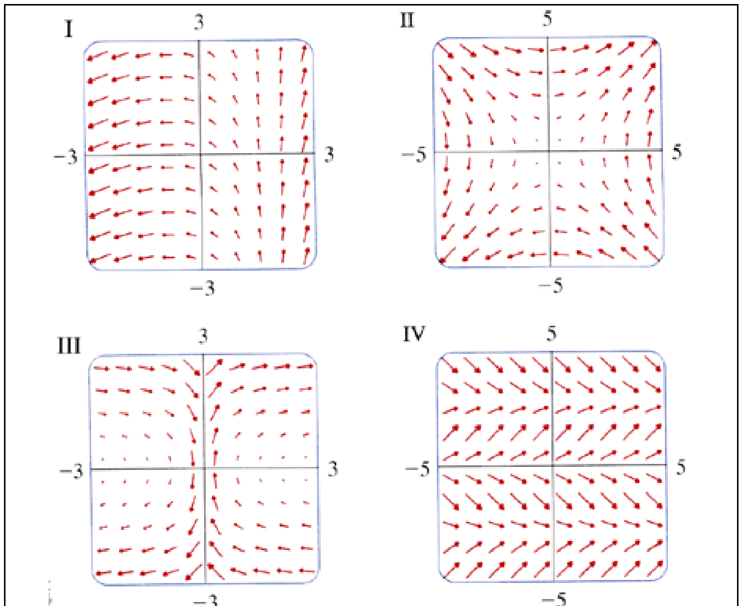
In problems 11-14, match the vector fields **F** with the plots labeled I-IV. Give reasons for your choices.

11. $\mathbf{F}(x,y) = \langle y, x\rangle$

12. $\mathbf{F}(x,y) = \langle 1, \sin y\rangle$

13. $\mathbf{F}(x,y) = \langle x-2, x+1\rangle$

14. $\mathbf{F}(x,y) = \langle y, \frac{1}{x}\rangle$



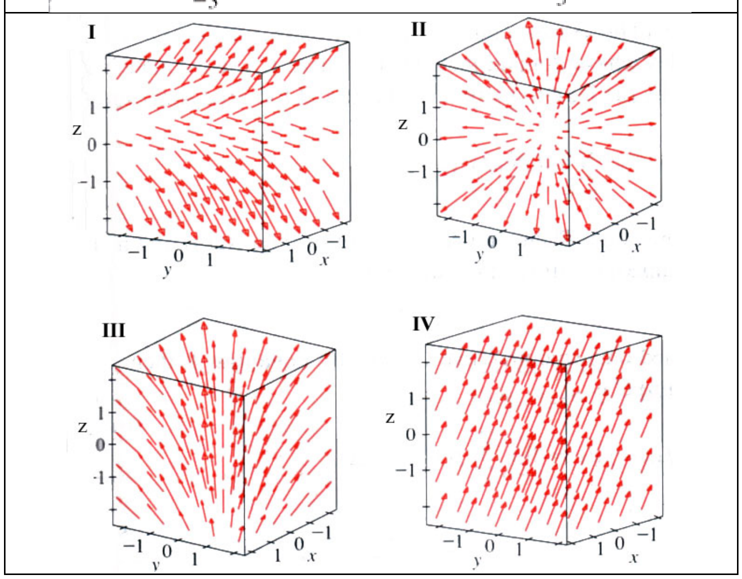
In problems 15-18, match the vector fields **F** on $\mathbf{R}^3$ with the plots labeled I-IV. Give reasons for your choices.

15. $\mathbf{F}(x,y,z) = \mathbf{i} + 2\,\mathbf{j} + 3\mathbf{k}$

16. $\mathbf{F}(x,y,z) = \mathbf{i} + 2\,\mathbf{j} + z\,\mathbf{k}$

17. $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + 3\mathbf{k}$

18. $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$



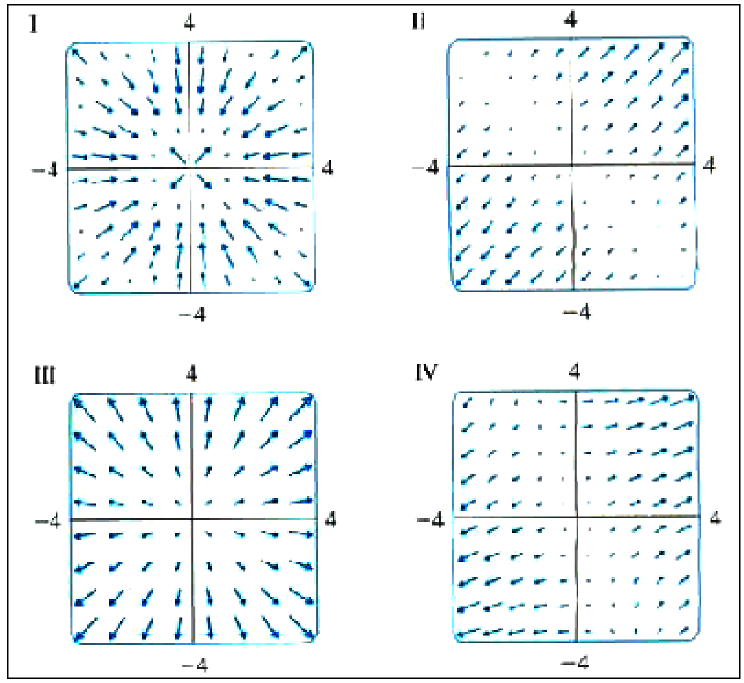
In problems 29-32, match the functions *f* with the plots of their gradient vector fields (labeled I-IV). Give reasons for your choices.

29. $f(x, y) = x^2 + y^2$

30. $f(x, y) = x(x + y)$

31. $f(x, y) = (x + y)^2$

32. $f(x, y) = \sin\left(\sqrt{x^2 + y^2}\right)$



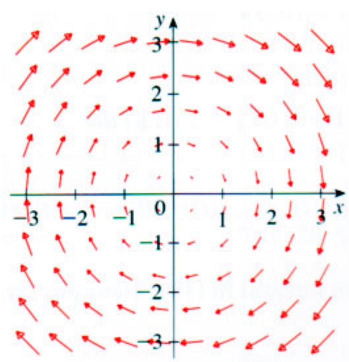
Section 13.2:

1. Evaluate the line integral where C is the given curve: $\int_C y^3\, ds \quad C: x = t^3, y = t, 0 \le t \le 2$.

14. Evaluate the integral $\int_C z\,dx + x\,dy + y\,dz$ where C is given by $x = t^2,\ y = t^3,\ z = t^2,\ 0 \le t \le 1$.

15. Evaluate the integral $\int_C (x + yz)\,dx + 2x\,dy + xyz\,dz$ where C consists of line segments from (1,0,1) to (2,3,1) and from (2,3,1) to (2,5,2).

17. Let **F** be the vector field shown in the figure.
 a) If C_1 is the vertical line segment from $(-3,-3)$ to $(-3,3)$, determine whether $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative or zero.
 b) If C_2 is the counterclockwise-oriented circle with radius 3 and center the origin, determine whether $\int_{C_2} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative or zero.



27. Use a graph of the vector field **F** and the curve C to guess whether the line integral of **F** over C is positive, negative, or zero. Then evaluate the line integral: $\mathbf{F}(x, y) = (x - y)\,\mathbf{i} + xy\,\mathbf{j}$, C is the arc of the circle $x^2 + y^2 = 4$ traversed counterclockwise from $(2,0)$ to $(0,-2)$.

Section 13.3:

19. Show that the line integral is independent of path and evaluate the integral.

$$\int_C \tan y\,dx + x\sec^2 y\,dy, \quad C \text{ is any path from } (1,0) \text{ to } (2, \tfrac{\pi}{4}).$$