Math E-21a – Fall 2018 – HW #10 problems

Problems to turn in on Thurs, Nov 15 (or online no later than Sat, Nov 17, 11:59pm):
Section 12.7:

12. Evaluate the triple integral $\iiint_E y\,dV$ where E is bounded by the planes $x=0$, $y=0$, $z=0$, and $2x+2y+z=4$.

16. Evaluate the triple integral $\iiint_T xyz\,dV$ where T is the solid tetrahedron with vertices at $(0,0,0)$, $(1,0,0)$, $(1,1,0)$, and $(1,0,1)$.

18. Evaluate the triple integral $\iiint_E z\,dV$ where E is bounded by the cylinder $y^2+z^2=9$ and the planes $x=0$, $y=3x$, and $z=0$ in the first octant (i.e. where *x*, *y*, and *z* are all nonnegative).

22. Use a triple integral to find the volume of the solid enclosed by the paraboloid $x=y^2+z^2$ and the plane $x=16$.

38. Find the mass and center of mass of the solid E bounded by the parabolic cylinder $z=1-y^2$ and the planes $x+z=1$, $x=0$, and $z=0$ with the density function $\delta(x,y,z)=4$.

46. Set up, but do not evaluate, integral expressions for (a) the mass, (b) the center of mass, and (c) the moment of inertia about the *z*-axis for the solid hemisphere $x^2+y^2+z^2\le 1, \quad z\ge 0$; with density function $\delta(x,y,z)=\sqrt{x^2+y^2+z^2}$.

Section 12.8:

10. Use cylindrical coordinates to evaluate $\iiint_E x\,dV$, where E is the solid enclosed by the planes $z=0$, $z=x+y+5$ and by the cylinders $x^2+y^2=4$ and $x^2+y^2=9$.

12. Find the volume that lies within both the cylinder $x^2+y^2=1$ and the sphere $x^2+y^2+z^2=4$.

16. Find the mass of a ball B given by $x^2+y^2+z^2\le a^2$ if the density at any point is proportional to its distance from the *z*-axis.

26. Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2+y^2+z^2=4$, above the *xy*-plane, and below the cone $z=\sqrt{x^2+y^2}$.

28. Let H be a solid hemisphere of radius a where the density at any point is proportional to the distance from the center of the base.
   (a) Find the mass of H.
   (b) Find the center of mass of H.
   (c) Find the moment of inertia of H about its axis.

31. Use cylindrical or spherical coordinates, whichever seems more appropriate, to find the volume and centroid of the solid E that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$.

For additional practice:
Section 12.7:

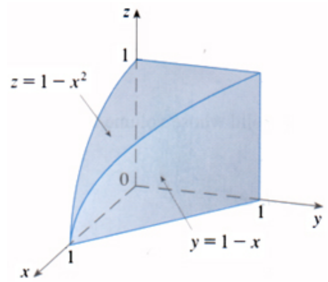
4. Evaluate the iterated integral: $\int_0^1 \int_x^{2x} \int_0^y 2xyz\, dz\, dy\, dx$

11. Evaluate the triple integral $\iiint_E 6xy\, dV$ where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

17. Evaluate the triple integral $\iiint_E x\, dV$ where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

21. Use a triple integral to find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$.

34. The figure shows the region of integration for the integral $\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)\, dy\, dz\, dx$. Rewrite this integral as an equivalent iterated integral in the five other possible orders of integration.



37. Find the mass and center of mass of the solid E of Exercise 9 (under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$) with the density function $\delta(x, y, z) = 2$.

45. Set up, but do not evaluate, integral expressions for (a) the mass, (b) the center of mass, and (c) the moment of inertia about the z-axis for the solid of Exercise 19 (enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$) with density function $\delta(x, y, z) = \sqrt{x^2 + y^2}$.

51. Find the average value of the function $f(x, y, z) = xyz$ over the cube with side length L that lies in the first octant with one vertex at the origin and edges parallel to the coordinate axes.

Section 12.8:

7. Use cylindrical coordinates to evaluate $\iiint_E \sqrt{x^2 + y^2}\, dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

11. Use cylindrical coordinates to evaluate $\iiint_E x^2\, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

13. Use cylindrical coordinates to (a) find the volume of the region E bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$; and (b) find the centroid of the region E in part (a).

19. Evaluate $\iiint_E z\, dV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

35. Evaluate the integral by changing to cylindrical coordinates: $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\, dz\, dx\, dy$.

37. Evaluate the integral by changing to spherical coordinates: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\, dy\, dx$.