Math E-21a – Fall 2011 – HW #1 problems

Note: Points will be typically denoted using ordinary parentheses ( ), vectors using angled brackets $\langle\ \rangle$.

Due in class on Thurs, Sept 8:

Section 9.1:

8. Find the distance from (3, 7,–5) to each of the following.
   (a) The *xy*-plane (b) The *yz*-plane (c) The *xz*-plane
   (d) The *x*-axis (e) The *y*-axis (f) The *z*-axis

12. Find an equation of the sphere that passes through the origin and whose center is $(1,2,3)$.

14. Show that the equation $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$ represents a sphere, and find its center and radius.

16. Show that the equation $3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$ represents a sphere, and find its center and radius.

38. Consider the points *P* such that the distance from *P* to *A* $(-1,5,3)$ is twice the distance from *P* to *B* $(6,2,-2)$. Show that the set of all such points is a sphere, and find its center and radius.

Section 9.2:

22. Find a vector that has the same direction as $\langle -2,4,2\rangle$ but has length 6.

28. The magnitude of a velocity vector is called *speed*. Suppose that a wind is blowing from the direction N45°W at a speed of 50 km/h. (This means that the direction from which the wind blows is 45° west of the northerly direction.) A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 250 km/h. The *true course*, or *track*, of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The *ground speed* of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.

38. Suppose a vector **a** makes angles α, β, and γ with the positive *x*-, *y*-, and *z*-axes, respectively. Find the components of **a** and show that
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
(The numbers cos α, cos β, and cos γ are called the *direction cosines* of **a**.)

43. Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

Challenge Problem: Use vectors to show that the medians of a triangle intersect at a point one-third of the way along each median from the side it bisects.

For additional practice (not to be turned in):

Section 9.1:

7. Find the lengths of the side of the triangle PQR. Is it a right triangle? Is it an isosceles triangle?
   (a) $P(3,-2,-3)$, $Q(7,0,1)$, $R(1,2,1)$ (b) $P(2,-1,0)$, $Q(4,1,1)$, $R(4,-5,4)$

11. Find an equation of the sphere that passes through the point $(4,3,-1)$ and has center $(3,8,1)$.

In problems 21-32, describe in words the region in $\mathbf{R}^3$ represented by the equation or inequality.

21. $x = 5$ 22. $y = -2$ 23. $y < 8$ 24. $x > -3$ 25. $0 \le z \le 6$ 26. $z^2 = 1$ 27. $x^2 + y^2 = 4,\ z = -1$

28. $y^2 + z^2 = 16$ 29. $x^2 + y^2 + z^2 \le 3$ 30. $x = z$ 31. $x^2 + z^2 \le 9$ 32. $x^2 + y^2 + z^2 > 2z$

Section 9.2:

In problems 15-18, find $\mathbf{a}+\mathbf{b}$, $2\mathbf{a}+3\mathbf{b}$, $\|\mathbf{a}\|$, and $\|\mathbf{a}-\mathbf{b}\|$. [Here $\|\mathbf{a}\|$ denotes the magnitude of the vector **a**.]

15. $\mathbf{a} = \langle 5,-12\rangle,\quad \mathbf{b} = \langle -3,-6\rangle$ 16. $\mathbf{a} = 4\mathbf{i} + \mathbf{j},\quad \mathbf{b} = \mathbf{i} - 2\mathbf{j}$

17. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k},\quad \mathbf{b} = -2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ 18. $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k},\quad \mathbf{b} = 2\mathbf{j} - \mathbf{k}$