Problem Set #9 – Concourse 18.03 – Spring 2019
(due Thurs, May 9 in 16-137)

Problem 1: Write the following equations as equivalent first-order systems.

a) $\dfrac{d^2x}{dt^2}+5\dfrac{dx}{dt}+tx^2=0$ b) $y''-x^2y'+(1-x^2)y=\sin x$

Problem 2: (a) Find the coordinates of the vector $\mathbf{x}=\begin{bmatrix}7\\2\end{bmatrix}$ relative to the basis $\mathcal{B}=\left\{\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}2\\5\end{bmatrix}\right\}$.

(b) Find the coordinates of the vector $\mathbf{x}=\begin{bmatrix}7\\2\\3\end{bmatrix}$ relative to the basis $\mathcal{B}=\left\{\begin{bmatrix}2\\1\\3\end{bmatrix},\begin{bmatrix}1\\1\\-1\end{bmatrix},\begin{bmatrix}0\\1\\-3\end{bmatrix}\right\}$.

In the next three problems, solve the given DE system $\frac{d\mathbf{x}}{dt}=\mathbf{A}\mathbf{x}$. First find the eigenvalues and associated eigenvectors, and from these construct the normal modes and thus the general solution. In addition, provide a diagram (either by hand or by using the Java tool to draw the vector field and some trajectories).

Problem 3: Solve $\frac{d\mathbf{x}}{dt}=\mathbf{A}\mathbf{x}$, where $\mathbf{A}=\begin{bmatrix}-3&4\\-2&3\end{bmatrix}$ with initial conditions $\mathbf{x}(0)=\begin{bmatrix}7\\2\end{bmatrix}$.

Problem 4: Solve $\frac{d\mathbf{x}}{dt}=\mathbf{A}\mathbf{x}$ where $\mathbf{A}=\begin{bmatrix}4&-3\\8&-6\end{bmatrix}$ with initial conditions $\mathbf{x}(0)=\begin{bmatrix}7\\2\end{bmatrix}$.

Problem 5: Solve $\frac{d\mathbf{x}}{dt}=\mathbf{A}\mathbf{x}$ where $\mathbf{A}=\begin{bmatrix}1&-1&0\\1&2&1\\-2&1&-1\end{bmatrix}$ with initial conditions $\mathbf{x}(0)=\begin{bmatrix}7\\2\\3\end{bmatrix}$.

Problem 6: Find the real solutions to the system $\frac{d\mathbf{x}}{dt}=\mathbf{A}\mathbf{x}$ where $\mathbf{A}=\begin{bmatrix}3&-4\\4&3\end{bmatrix}$.

Problem 7: Solve the system $\dfrac{d\mathbf{x}}{dt}=\begin{bmatrix}-1&-2\\2&-1\end{bmatrix}\mathbf{x}$ with $\mathbf{x}(0)=\begin{bmatrix}1\\-1\end{bmatrix}$. Give the solution in real form. Sketch the solution.

Problem 8: Solve the differential equation $\ddot{x}+3\dot{x}-4x=0,\ \ x(0)=5,\ \dot{x}(0)=2$ two ways:

(a) using previous (non-matrix) methods.

(b) by reduction of order, turning this into a system of two 1st order ODEs, and solving using eigenvalue/eigenvector methods. Draw the phase plane with a range of sample trajectories, including the one associated with the given initial conditions.

Problem 9: Solve the differential equation $\ddot{x}+4\dot{x}+5x=0,\ \ x(0)=6,\ \dot{x}(0)=4$ two ways:

(a) using previous (non-matrix) methods.

(b) by reduction of order, turning this into a system of two 1st order ODEs, and solving using eigenvalue/eigenvector methods. Draw the phase plane with a range of sample trajectories, including the one associated with the given initial conditions.