Problem Set #8 – Concourse 18.03 – Spring 2019
(due Tuesday, April 30 in 16-137)

Problem 1: For each of the following functions, find the LTI (linear, time invariant, i.e. constant coefficients) differential operator $p(D)$ having it as unit impulse response.

(i) $2u(t)$. (ii) $u(t)t$. (iii) $u(t)t^2$.

Problem 2: Find $\mathcal{L}^{-1}\left(\dfrac{3s^3-3s^2+s-39}{s^4+4s^3+13s^2}\right)$

Problem 3: Solve the following IVP by using Laplace transform methods:

$$y''-2y'+2y=2e^t, \quad y(0-)=0, \quad y'(0-)=1$$

Problem 4: Let a and b be real numbers, with $a\neq 0$. Solve $a\dot{x}+bx=t$ with rest initial conditions in three ways:

(a) Undetermined coefficients to get x_p, and add the appropriate transient.

(b) Apply Laplace transform, solve, and transform back.

(c) Compute $w(t)*t$ (same as $t*w(t)$) using the weight function $w(t)$.

Note: In each of these approaches, don't forget to handle the $b=0$ case separately.

Problem 5: Solve the IVP: $\ddot{x}+2\dot{x}+x=3e^{-2t}\sin t, \quad x(0)=2, \quad \dot{x}(0)=1$ in three ways:

(a) By finding homogeneous solutions, a particular solution, and linearity (i.e., without Laplace transform)

(b) By applying the Laplace transform to both sides of the given ODE, incorporating the initial conditions.

(c) By finding the unit impulse response, i.e. the weight function $w(t)$ (with rest initial conditions), using convolution to find the zero state response (ZSR), finding the zero input response (ZIR), and combining these to give the solution.

Note: The convolution may require some integration by parts or you may use the following integral formulas:

$$\int e^{au}\sin bu\,du=\tfrac{1}{a^2+b^2}e^{au}(a\sin bu-b\cos bu)$$

$$\int ue^{au}\sin bu\,du=\tfrac{1}{a^2+b^2}ue^{au}(a\sin bu-b\cos bu)-\tfrac{1}{(a^2+b^2)^2}e^{au}[(a^2-b^2)\sin bu-2ab\cos bu]$$

[*You might want to try deriving this just for some extra fun.*]