Problem Set #7 – Concourse 18.03 – Spring 2019
(due Thurs, April 18 in 16-137)

Generalized Functions [Step and delta functions]

Problem 1: For each of the following functions $f(t)$, (i) draw a graph, (ii) draw a graph of the generalized derivative, (iii) write a single formula, if possible, for $f(t)$ and for $f'(t)$ (with possibly a few values not defined) using step functions $u(t-a)$, delta functions $\delta(t-a)$, and other (regular) functions.

(a) $f(t)=0$ for $t<0$, $f(t)=-t$ for $t>0$.

(b) $f(t)=0$ for $t<0$, $f(t)=1-t$ for $t>0$.

(c) $f(t)=0$ for $t<0$, $f(t)=2t-1$ for $0<t<1$, $f(t)=0$ for $t>1$.

(d) $f(t)=0$ for $t<0$, $f(t)=t-\lfloor t \rfloor$ for $t>0$, where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t.

Step and Delta Responses

[Note: The unit impulse response is the response associated with the standard delta function $\delta(t)$ and the unit step response is the response associated with the Heaviside function $u(t)$, both with rest initial conditions.]

In Problems 1 and 2, find the unit impulse response $w(t)$ and the unit step response $v(t)$ for

Problem 2: (a) Find the unit impulse response $w(t)$ and the unit step response $v(t)$ for $D+kI$.

(b) Find the unit impulse response $w(t)$ and the unit step response $v(t)$ for $D^2+\omega_0{}^2 I$.

Problem 3: (a) Find the unit impulse response $w(t)$ for the LTI operator $2D^2+4D+4I$.

(b) Find the unit step response $v(t)$ for the same operator.

(c) Verify that $\dot{v}=w$ (as it should be, since $\dot{u}=\delta$).

Laplace Transform

Problem 4: By using the table of formulas, find:

(a) $\mathcal{L}(e^{-t}\sin 3t)$ (b) $\mathcal{L}(e^{2t}(t^2-3t+2))$.

Problem 5: Find $\mathcal{L}(e^{-t}\sin 3t)$ by writing $e^{-t}\sin 3t$ as a linear combination of complex exponentials. Compare the answer to that obtained in the previous problem.

Problem 6: Find $\mathcal{L}(t^4 e^{\pi t})$

Problem 7: Find the Laplace transform of $f(t)=(u(t)-u(t-2\pi))\sin(t)$ by use of the t-shift rule.

Laplace Transforms, Inverse Laplace Transform, and applications to solving ODEs

Problem 8: Find $\mathcal{L}^{-1}(\frac{3}{2s-4})$

Problem 9: Find $\mathcal{L}^{-1}(\frac{5s-6}{s^2-3s})$

In Problems 10 and 11, solve the following IVP's by using Laplace transform methods.

Problem 10: $y'-y=e^{3t}$, $y(0-)=1$

Problem 11: $y''+4y=\sin t$, $y(0-)=1$, $y'(0-)=0$