

MIT Concourse 18.03 – Spring 2019 – Problem Set #5

(due Thurs, March 21, 2017 in 16-137)

**Problem 1:** Find all solutions to the following homogeneous ODEs:

a)  $\frac{d^5x}{dt^5} - 4\frac{d^4x}{dt^4} + 4\frac{d^3x}{dt^3} = 0$

b)  $\frac{d^3x}{dt^3} + 6\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 8x = 0$

**Problem 2.** Find a particular solution to the differential equation  $y'' - 2y' + 4y = e^x \cos x$ . Use complex exponentials where possible.

**Problem 3.** Find a particular solution to the differential equation  $y'' - 6y' + 9y = e^{3x}$ .

**Problem 4:** A resistor of 12 ohms is connected in series with an inductor of one henry, a capacitor of 0.01 farads, and a voltage source supplying  $12 \cos 10t$  volts. [This is a consistent system of units of measurement.] At  $t = 0$ , the charge on the capacitor is zero and the current in the circuit is also zero.

a) Determine  $Q(t)$ , the charge on the capacitor as a function of time for  $t > 0$ .

b) Determine  $I(t)$ , the current in the circuit as a function of time for  $t > 0$ .

**Problem 5:** Find a particular solution to the ODE  $\ddot{x} + 9x = 2 \sec 3t$ .

**Problem 6:** Find a particular solution to the ODE  $\ddot{x} + 9x = e^{-t}(t^2 + 1)$ .

**Problem 7:** Find a particular solution to the ODE  $\ddot{x} + 2\dot{x} + x = e^{-t} \ln t$  (where  $t > 0$ ) in two ways:

a) using the Exponential Shift Formula.

b) using Variation of Parameters.

**Problem 8:** Find a particular solution to the ODE  $t^2\ddot{x} + t\dot{x} - x = 72t^5$  given that  $x_1(t) = t$  and  $x_2(t) = \frac{1}{t}$  are homogeneous solutions.

**Problem 9.** A driven mass-spring-dashpot system is modeled by the DE:  $m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$  with  $m = 1$ ,  $c = 6$ ,  $k = 45$ , and  $F_0 = 50$ . Find the amplitude  $A(\omega)$  of the response as a function of the input frequency  $\omega$  and find the frequency which gives the largest system response.