MIT Concourse 18.03 – Spring 2019 – Problem Set #4

(due Thurs, Mar 14, 2019)

Problem 1. For the matrix $\mathbf{A}=\begin{bmatrix}1&3&6\\1&2&5\\1&1&4\end{bmatrix}$, find a basis of the image of **A** and a basis of the kernel of **A** and their dimensions.

Problem 2. For the matrix $\mathbf{A}=\begin{bmatrix}1&2&1&1&1\\2&2&4&3&1\\1&2&5&3&-1\\2&4&6&4&0\end{bmatrix}$, find a basis of the image of **A** and a basis of the kernel of **A** and their dimensions.

Problem 3. Consider the linear space $V=P_3=\{\text{polynomials of degree}\le 3\}$. <u>Find a basis</u> for the subspace W of V consisting of all polynomials $f(t)$ in P_3 such that $f(1)=0$ and $\int_{-1}^{1} f(t)\,dt=0$. [This is a subspace, i.e. it is closed under scaling and adding.]

Problem 4. The transformation $[T(f)](t)=f''(t)+4f'(t)$ from $P_2=\{\text{polynomials of degree}\le 2\}$ to P_2 is linear.

a) Is T an isomorphism, i.e. is it one-to-one and onto its codomain?

b) Find a basis for the kernel and the nullity of this transformation.

c) Find a basis for the image of this transformation and its rank.

Problem 5. Find the kernel and nullity of the transformation $T(f)=f-f'$ from C^∞ to C^∞.

[C^∞ denotes the linear space consisting of all infinitely differentiable functions of one variable.]

Problem 6. For the equation $y''+2y'+cy=0$, c constant, if we seek exponential solutions $y=e^{rt}$ and examine the resulting characteristic polynomial:

a) Tell which values of c correspond to each of the three cases: two real roots, repeated real root, and complex roots.

b) For the case of two real roots, tell for which values of c both roots are negative, both roots are positive, or the roots have different signs.

c) Summarize the above information by drawing a c-axis, and marking the intervals on it corresponding to the different possibilities for the roots of the characteristic equation.

d) Finally, use this information to mark the interval on the c-axis for which the corresponding ODE is stable. (The stability criterion using roots is what you will need.)

e) Specifically, solve the initial value problem: $y''+2y'-3y=0,\quad y(0)=1,\quad y'(0)=-1$.

f) Specifically, solve the initial value problem: $y''+2y'+5y=0,\quad y(0)=1,\quad y'(0)=-1$.

g) Specifically, solve the initial value problem: $y''+2y'+y=0,\quad y(0)=1,\quad y'(0)=-1$.

In the next several problems, find the general solution to the given ODE and also the specific solution satisfying the given initial conditions.

Problem 7. $2y''-3y'=0,\quad y(0)=3,\quad y'(0)=1$

Problem 8. $y''-6y'+25y=0,\quad y(0)=3,\quad y'(0)=1$.

Problem 9. $y''+2y'-3y=2t-5,\quad y(0)=1,\quad y'(0)=-1$

Problem 10. Find a particular solution to the ODE: $\ddot{x}+x=t^2+\cos(2t-1)$