MIT Concourse 18.03 – Problem Set #2

(due Thurs, Feb 21, 2019 at 4:00pm in 16-137)

In Problems 1-3, solve the given linear DE. Give the general solution, and also the specific solution satisfying the initial condition. Note: $y' = \frac{dy}{dx}$

Problem 1: (EP 1.5/6) $xy' + 5y = 7x^2, \quad y(2) = 5$

Problem 2: (EP 1.5/16) $y' = (1-y)\cos x, \quad y(\pi) = 2$

Problem 3: (EP 1.5/20) $y' = 1 + x + y + xy, \quad y(0) = 0$

Problem 4: (EP 1.5/28) Solve the differential equation $(1+2xy)\frac{dy}{dx} = 1 + y^2$ by regarding *y* as the independent variable and *x* as the dependent variable. Express your solution as $x = x(y)$. [This problem may test your integration skills!]

Problem 5: (EP 1.5/29) Express the general solution of the differential equation $\frac{dy}{dx} = 1 + 2xy$ in terms of the **error function** $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$.

Problem 6: (EP 1.5/32 - modified) Find the solution of the differential equation $\frac{dy}{dx} + y = 2\sin x, \; y(0) = 1$ two ways:

(a) by finding an appropriate integrating factor and carrying out the necessary integrations and algebra.

(b) by finding a solution to the corresponding homogeneous problem, finding a particular solution to the inhomogeneous problem, and using linearity principles. [Try a particular solution of the form $y_p(x) = A\sin x + B\cos x$.]

(c) Reformulate your solution to this initial value problem in the form $y(x) = A\cos(\omega x - \phi_0) + ce^{-kx}$ for appropriate constants *A* (amplitude), ω (frequency), ϕ_0 (phase angle), *c*, and *k*.

Problem 7: (EP 1.5/41) A 30-year-old woman accepts an engineering position with a starting salary of $30,000 per year. Her salary $S(t)$ increases exponentially, with $S(t) = 30e^{t/20}$ thousand dollars after *t* years. Meanwhile, 12% of her salary is deposited continuously in a retirement account, which accumulates interest at a continuous annual rate of 6%.

(a) Estimate ΔA in terms of Δt to derive the differential equation satisfied by the amount $A(t)$ in her retirement account after *t* years.

(b) Compute $A(40)$, the amount available for her retirement at age 70.