Concourse 18.02 Problem Set 8 – Fall 2018

due Thurs, Nov 8

Read sections 14.6 (Triple Integrals), 14.7 (Integration in Cylindrical and Spherical Coordinates), 14.8 (Surface Area), 14.9 (Change of Variables in Multiple Integrals); and Supplementary Notes I (Limits in Iterated Integrals), CV (Changing Variables in Multiple Integrals), and G (Gravitational Attraction); and do the following problems:

1. [14.6/14] Sketch the solid bounded by the graphs of the equations $z = x^2 + y^2$, $z = 0$, $x = 0$, $y = 0$, and $x + y = 1$. Then find its volume by triple integration.

2. [14.6/31] Consider the solid paraboloid bounded by $z = x^2 + y^2$ and the plane $z = h > 0$. Show that its centroid lies on its axis of symmetry, two-thirds of the way from its "vertex" $(0,0,0)$ to its base.

3. [14.7/4] Find the moment of inertia around the *z*-axis of a spherical ball of radius *a* given that the *z*-axis passes through its center and the solid has uniform density and total mass *M*.

4. [14.7/5] Find the volume of the region that lies inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$.

5. [14.7/16] A homogeneous solid cylinder has mass *m* and radius *a*. Show that its moment of inertia around its axis of symmetry is $\frac{1}{2}ma^2$.

6. [14.7/35] Suppose a gaseous spherical star of radius *a* has density function $\delta = k(1-\rho^2/a^2)$, so its density varies from $\delta = k$ at its center to $\delta = 0$ at its boundary $\rho = a$. Show that its mass is $\frac{2}{5}$ that of a similar star with uniform density *k*.

7. [14.7/39] Find the average distance of points of a solid ball of radius *a* from the center of the ball.

8. [14.8/8] Find the area of the ellipse that is cut from the plane $2x + 3y + z = 6$ by the cylinder $x^2 + y^2 = 2$.

9. [14.8/10] Find the area that is cut from the surface $z = x^2 - y^2$ by the cylinder $x^2 + y^2 = 4$.

10. [14.9/7] Let *R* be the parallelogram bounded by the lines $x + y = 1$, $x + y = 2$, and $2x - 3y = 2$, $2x - 3y = 5$. Substitute $u = x + y$, $v = 2x - 3y$ to find its area $A = \iint_R 1\,dx\,dy$.

11. [14.9/8] Substitute $u = xy$, $v = y/x$ to find the area of the first-quadrant region bounded by the lines $y = x$, $y = 2x$ and the hyperbolas $xy = 1$, $xy = 2$.

12. [14.9/14] Let *R* be the solid ellipsoid with outer boundary surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Use the transformation $x = au$, $y = bv$, $z = cw$ to show that the volume of this ellipsoid is $V = \iiint_R 1\,dx\,dy\,dz = \frac{4}{3}\pi abc$.

13. [SN-3D/1] Evaluate $\iint_R \left(\frac{x-3y}{2x+y}\right) dxdy$, where *R* is the parallelogram bounded on the sides by $y = -2x + 1$, $y = -2x + 4$, and above and below by $y = x/3$ and $y = (x-7)/3$.
    Use a change of variables $u = x - 3y$, $v = 2x + y$.

14. [SN-3D/3] Find the volume underneath the surface $z = 16 - x^2 - 4y^2$ and over the *xy*-plane; simplify the integral by making the change of variable $u = x$, $v = 2y$.

Note: In the change of variables problems, you may want to approach them by ignoring the suggested change of variables and just use the geometry of the domain to determine the ideal new variables.