Concourse 18.02 Problem Set 4 – Fall 2018

due Friday, October 5

Read sections 12.7 (Quadric Surfaces), 13.1-13.2 (Functions of Several Variables), 13.3 (Limits and Continuity), 13.4 (Partial Derivatives); 13.6 (Increments and Linear Approximation); and Supplementary Notes TA (Tangent Approximation) and do the following problems:

Quadric Surfaces:
In each of the following problems, (**briefly**) describe and sketch the graphs of the equations given in the following problems. [Note that both of these equations represent surfaces in $\mathbf{R}^3$.]

1. [12.7/8] $4x^2 + 9y^2 = 36$ 2. [12.7/26] $x^2 - y^2 - 9z^2 = 9$

Functions of Several Variables

3. [13.2/12] State the largest possible domain of definition of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$. [In stating the largest possible domain, drawing it as a region in the *xy*-plane is a good idea.]

In each of the following problems, describe the graph of the function *f*. [Draw a few cross-sections.]

4. [13.2/26] $f(x, y) = 4 - x^2 - y^2$ 5. [13.2/29] $f(x, y) = 10 - \sqrt{x^2 + y^2}$

In each of the following problems, sketch some typical level curves of the function *f*.

6. [13.2/33] $f(x, y) = x^2 + 4y^2$ 7. [13.2/34] $f(x, y) = y - x^2$

Limits and Continuity:
In each of the following problems, investigate the existence of the given limit by making the substitution $y = mx$. [That is, calculate the limits as you approach $(0,0)$ along several lines passing through $(0,0)$.]

8. [13.3/43] $\lim_{(x,y)\to(0,0)} \left(\dfrac{x^2 - y^2}{x^2 + y^2} \right)$ 9. [13.3/44] $\lim_{(x,y)\to(0,0)} \left(\dfrac{x^4 - y^4}{x^4 + x^2y^2 + y^4} \right)$

10. Show that $\lim_{(x,y)\to(0,0)} \left(\dfrac{x^2y^3}{x^2 + y^2} \right) = 0$. [*Hint*: Relate the algebraic expressions to the distance from the origin using equalities and inequalities. Then show that as the distance from the origin decreases to 0, the values of the given function necessarily also decrease to 0.]

Partial Derivatives:
In each of the following problems, compute the first-order partial derivatives of each function.

11. [13.4/6] $f(x, y) = \dfrac{xy}{x^2 + y^2}$ 12. [13.4/10] $f(x, y) = \tan^{-1}(xy)$

13. [13.4/40] Find an equation of the plane tangent to the surface $z = \sqrt{x^2 + y^2}$ at the point $P = (3, -4, 5)$.

14. a) Find the curve of intersection of the surfaces $z = x^2 - y^2$ and $z = 2 + (x - y)^2$ in parametric form.

b) Find the angle of intersection of these two surfaces at the point $(2,1,3)$. (The angle of intersection of two surfaces is defined to be the angle made by their tangent planes (or the acute angle between normal vectors to their respective tangent planes.)

c) Check that the tangent vector to the curve of intersection found in part (a) at the point $(2,1,3)$ lies in (i.e. is parallel to) the tangent plane of each of the two surfaces.

Increments and Linear Approximation:

15. [13.6/25] Use differentials to approximate $\left(\sqrt{15}+\sqrt{99}\right)^2$.

16. [13.6/34] The base radius *r* and the height *h* of a right circular cylinder are measured as 3 cm and 9 cm, respectively. There is a possible error of 1 mm in each measurement. Use differentials to estimate the maximum possible error in computing: (a) the volume of the cylinder; (b) the total surface area of the cylinder.

17. [SN-2B/4] The combined resistance R of two wires in parallel, having resistances R_1 and R_2 respectively, is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$. If the resistances in the wires are initially 1 and 2 ohms, with a possible error in each of ±0.1 ohm, what is the value of *R*, and by how much might this be in error? (Use the approximation formula.)

18. [SN-2C/3] Two sides of a triangle have lengths respectively *a* and *b*, with θ the included angle. Let *A* be the area of the triangle.

    a) Express *dA* in terms of the variables and their differentials.

    b) If $a=1,\ b=2,\ \theta=\frac{\pi}{6}$, to which variable is *A* most sensitive? least sensitive?

    c) Using the values in (b), if the possible error in each value is 0.02, what is the possible error in *A*, to two decimal places?