Concourse 18.02 Problem Set 10 – Fall 2011

Not to be turned in, but definitely do them!

Read sections:

15.7 (Stokes' Theorem)
Notes V13 (Stokes' Theorem)

Notes V14 (Some Topological Questions)

Notes V15 (Relations to Physics)

Part I

Recommended Problems:

Stokes' Theorem
15.7/1,9
SN-6F/1b,2,3,5

Topological Questions
SN-6G/1

Relations to Physics
SN-6H/2

Additional Practice Problems:

Stokes' Theorem
15.7/1,3,5,7,19
SN-6F/1a

Topological Questions
SN-6G/2

Relations to Physics
SN-6H/1

Part II

Problem 1: Let $\mathbf{F}(x,y,z)=\left(\frac{-z}{x^2+z^2}\right)\mathbf{i}+y\,\mathbf{j}+\left(\frac{x}{x^2+z^2}\right)\mathbf{k}$ defined for all points (x,y,z) in 3-space not on the y-axis (that is, all points for which $x^2+z^2>0$.

a) By direct computation, show that $\nabla\times\mathbf{F}=\mathbf{0}$ for all points not on the y-axis.

b) By direct computation, show that $\oint_{C_1}\mathbf{F}\cdot d\mathbf{r}=0$ where C_1 is the closed curve defined by $x^2+y^2=1,\ z=1$.

c) Can you use Stokes' Theorem and the fact (from part (a)) that $\nabla\times\mathbf{F}=\mathbf{0}$ to conclude that $\oint_{C_2}\mathbf{F}\cdot d\mathbf{r}=0$ when C_2 is the closed curve defined by $x^2+z^2=1,\ y=0$? Why/ why not ?

d) Compute out $\oint_{C_2}\mathbf{F}\cdot d\mathbf{r}=0$ and see what happens.

Problem 2: Suppose the field **F** is replaced by the field $\mathbf{G}(x,y,z)=\left(\frac{x}{x^2+y^2+z^2}\right)\mathbf{i}+\left(\frac{y}{x^2+y^2+z^2}\right)\mathbf{j}+\left(\frac{z}{x^2+y^2+z^2}\right)\mathbf{k}$ defined for all points $(x,y,z)\neq(0,0,0)$.

a) Show that $\nabla\times\mathbf{G}=\mathbf{0}$ for all points $(x,y,z)\neq(0,0,0)$.

b) Can you use Stokes' Theorem in this case and the fact that $\nabla\times\mathbf{G}=\mathbf{0}$ for all points $(x,y,z)\neq(0,0,0)$ to conclude that $\oint_C\mathbf{F}\cdot d\mathbf{r}=0$ for all simple closed curves C which do not pass through the origin?

c) Explain the difference between these two cases in term of the connectedness type of the domains of definition of the two fields.