# Concourse 18.02 Problem Set 10 – Fall 2018

due Thurs, Nov 29

| **Read sections**: | |
|---|---|
| **Notes V3** (Two-dimensional Flux) | **15.5** (Surface Integrals) |
| **Notes V4** (Green's Theorem in Normal Form) | **Notes V9** (Surface Integrals) |
| **14.8** (Surface Area) | **15.6** (The Divergence Theorem) |

1. [SN-4E/3] Let $\mathbf{F} = x^2\,\mathbf{i} + xy\,\mathbf{j}$. Evaluate $\int_C \mathbf{F}\cdot\mathbf{n}\,ds$ if *C* is given by $\mathbf{r}(t) = (t+1)\,\mathbf{i} + t^2\,\mathbf{j}$, where $0 \le t \le 1$; the positive direction on *C* is the direction of increasing *t*.

2. [SN-4F/4] Verify Green's theorem in the normal form by calculating both sides and showing they are equal if $\mathbf{F} = x^2\,\mathbf{i} + xy\,\mathbf{j}$, and *C* is the square with opposite vertices at $(0,0)$ and $(1,1)$.

5. [15.5/5] Evaluate the surface integral $\iint_S (xy+1)\,dS$ where *S* is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.

6. [15.5/24] Calculate the outward flux of the vector field $\mathbf{F} = x^2\,\mathbf{i} + 2y^2\,\mathbf{j} + 3z^2\,\mathbf{k}$ across the closed surface *S* where *S* is the boundary of the solid cone bounded by $z = \sqrt{x^2+y^2}$ and the plane $z = 3$.

7. [15.5/25] The first-octant part of the spherical surface $\rho = a$ has unit density. Find its centroid.

8. [SN-6B/5] Find $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = z\,\mathbf{k}$, and S is the portion of the plane $x + y + z = 1$ lying in the first octant (take **n** pointed away from the origin).

9. [SN-6B/12] Find the average height above the *xy*-plane of a point chosen at random on the surface of the hemisphere $x^2 + y^2 + z^2 = a^2,\ z \ge 0$.

10. Show that the average straight-line distance to a fixed point on the surface of a sphere of radius *a* (averaged over the whole surface of the sphere) is $\frac{4a}{3}$. [<u>Suggestion</u>: take the sphere centered at the origin, and choose the "north pole" as the fixed point.]

11. [15.6/5] Verify the Divergence Theorem by direct computation of the surface integral and the triple integral if $\mathbf{F} = (x+y)\,\mathbf{i} + (y+z)\,\mathbf{j} + (z+x)\,\mathbf{k}$ and *S* is the surface of the tetrahedron bounded by the three coordinate planes and the plane $x + y + z = 1$.

12. Let *S* = *S*(*P*,*C*) be the portion of the `ruled surface' above the *xy*-plane and below *P* where the base curve *C* is the circle of radius 1 centered at (0,1) in the *xy*-plane, and the point *P* is at $(0,0,a)$. Thus *S* is a slant cone with circular base of radius 1 and height *a*.

    a) Find parametric equations of the surface *S*.

    Let **F** be the field given by $\mathbf{F}(x,y,z) = \langle x, y, z\rangle$. Find the upward flux of **F** through S in two ways:

    b) directly using a surface integral; and

    c) by completing *S* to a closed surface using the interior of *C* in the *xy*-plane, using the Divergence Theorem, and adjusting for the flux through the base as needed.

    d) How does the flux of **F** through S depend on *a*? Draw a rough sketch of both *S* and **F** on *S*, and then explain why this type of dependence on *a* makes sense (assuming that it does).