Concourse 18.01A Problem Set 6 – Fall 2018

(due Mon, Oct 22)

Topic: Infinite series, convergent sequences, harmonic series convergence tests.
Read: TB: 13.1 – 13.6.

Topic: Geometric series, ratio test, alternating series, absolute vs. conditional convergence.
Read: TB: 13.7 – 13.8.

Topic: Power Series; Taylor Series; Taylor's Theorem with Remainder.
Read: TB: 14.1 – 14.5.

For problems 1-8, determine if the given series is absolutely convergent, conditionally convergent, or divergent. You may use any test or other reasoning.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{n-1}{n^2+n}$

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n^2-1}}{n^3-2n^2+5}$

3. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+2}}\right)$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{2^{3n}}$

5. $\displaystyle\sum_{k=1}^{\infty}\frac{5^k}{3^k+4^k}$

6. $\displaystyle\sum_{n=1}^{\infty}\frac{3^n n^2}{n!}$

7. $\displaystyle\sum_{n=1}^{\infty}ne^{-n^2}$

8. $\displaystyle\sum_{n=1}^{\infty}(-1)^n 2^{1/n}$

By using the ratio test, determine the radius of convergence of each of the following power series.

9. [SN-7B/6b] $\displaystyle\sum_{n=1}^{\infty}\frac{2^n x^n}{n^2}$

10. [SN-7B/6e] $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+1}}{2^n\sqrt{n}}$

11. [SN-7B/6h] $\displaystyle\sum_{n=0}^{\infty}\frac{2^{2n} x^n}{n!}$

12. [SN-7C/2] Calculate $\sin 1$ using the Taylor series up to the term in x^5. Estimate the accuracy using the remainder term. (The calculator value is 0.84147.) Use the remainder term $R_6(x)$, not $R_5(x)$; why?

13. [SN-7C/3] Using the remainder term, tell for what value of *n* in the approximation $e^x \approx 1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}$ the resulting calculation will give *e* to 3 decimal places (by convention, this means: within .0005).

14. [SN-7C/4] By using the remainder term, tell whether $\cos x \approx 1-\frac{x^2}{2!}$ will be valid to within .001 over the interval $|x| < 0.5$.