Concourse 18.01A Problem Set 5 – Fall 2018

(due Monday, Oct 15)

Topic: Numerical Integration (briefly), esp. Trapezoid Rule, Midpoint Rule, Simpson's Rule.

Topic: Infinite series, convergent sequences, harmonic series convergence tests.
Read: TB: 13.1 – 13.6.

Topic: Geometric series, ratio test, alternating series, absolute vs. conditional convergence.
Read: TB: 13.7 – 13.8.

Find approximations to the following integrals using four intervals using Riemann sums with left endpoints, using the trapezoidal rule, and using Simpson's rule. Also give numerical approximations to the exact values of the integrals given to see how good these approximation methods are.

1. [SN-3G/1a] $\int_0^1 \sqrt{x}\,dx$

2. [SN-3G/1d] $\int_1^2 \frac{dx}{x}$ $(=\ln 2)$

Find the sum of the following geometric series:

3. [SN-6C/1a] $1+\frac{1}{5}+\frac{1}{25}+\cdots$

4. [SN-6C/1d] $0.4444\cdots$

5. [SN-6C/3] a) Use the upper and lower Riemann sums of $\ln n = \int_1^n \frac{dx}{x}$ to show that

$$\ln n < 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n} < 1+\ln n$$

b) Suppose that it takes 10^{-10} seconds for a computer to add one term in the series $\sum 1/n$. About how long would it take for the partial sum to reach 1000?

6. [SN-7A/1] Do the following series converge or diverge? Give reason. If the series converges, find its sum.

a) $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots+\frac{1}{4^n}+\cdots$ b) $1-1+1-1+\cdots+(-1)^n+\cdots$ c) $1+\frac{1}{2}+\frac{2}{3}+\cdots+\frac{n}{n+1}+\cdots$

d) $\ln 2+\ln\sqrt{2}+\ln\sqrt[3]{2}+\ln\sqrt[4]{2}+\cdots$ e) $\sum_{n=1}^{\infty}\frac{2^{n-1}}{3^n}$ f) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{3^n}$

Using the integral test, tell whether the following series converge or diverge; show work or reasoning.

7. [SN-7B/1a] $\sum_{n=0}^{\infty}\frac{n}{n^2+4}$

8. [SN-7B/1b] $\sum_{n=0}^{\infty}\frac{1}{n^2+1}$

9. [SN-7B/1d] $\sum_{n=1}^{\infty}\frac{\ln n}{n}$

Using the limit comparison test, tell whether each series converges or diverges; show work or reasoning. (For some of them, simple comparison works.)

10. [SN-7B/2c] $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+n}}$

11. [SN-7B/2e] $\sum_{n=0}^{\infty}\frac{\sqrt{n}}{n^2+1}$

12. [SN-7B/2f] $\sum_{n=1}^{\infty}\frac{\ln n}{n}$

Using the ratio test, or otherwise, determine whether or not each of these series is absolutely convergent. (Note that $0!=1$.)

13. [SN-7B/4b] $\sum_{n=0}^{\infty}\frac{2^n}{n!}$

14. [SN-7B/4f] $\sum_{n=1}^{\infty}\frac{n!}{n^n}$; use $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$