Mathematics 15a – Applied Linear Algebra – Fall 2007

Instructor: Robert Winters, **robert@math.rwinters.com** or **rwinters@brandeis.edu**. Office hours will be determined after consultation with the class – most like before the 1pm class and after the 2pm class on the days either class meets. My office is **Goldsmith 222**. My office phone number is 781-736-3054, but e-mail is preferred (since I check it more often).

Course website: http://math.rwinters.com/15a

Course meetings: Section 1 of the class meets Mondays, Wednesdays, and Thursdays from 1:10pm to 2:00pm in Goldsmith 117. Section 2 of the class meets Mondays and Wednesdays from 2:10pm to 3:30pm in Goldsmith 317.

Text: *Linear Algebra With Applications*, 3rd Edition, by Otto Bretscher, Prentice-Hall, 2005; ISBN 0-13-145334-3 for the text alone. It may also be purchased bundled with the student solutions book. We will cover most of the topics in this book, though some topics will be omitted due to time constraints. Homework will be assigned from its large (and excellent) collection of exercises. Earlier editions of the text are acceptable since assignments will be posted as PDFs. The international edition (paperback) may also be found online and its content is identical to the regular hardcover edition. A link is posted on the main page of the course website that searches for all online sources of the text and compares prices.

Recommended Prerequisites: One full year of single variable calculus. Though it is preferable to have previously taken multivariable calculus, this is not necessary for successful completion of this course. It will be very helpful if you have some familiarity with the algebra and geometry of lines and planes in $\mathbf{R}^2$, $\mathbf{R}^3$, and possibly $\mathbf{R}^n$, and the dot product of two vectors. You should be able to solve simple systems of equations and find the roots of polynomials. Also, it may be helpful if you can set up and solve simple differential equations.

Philosophy: This course is greatly dependent upon your participation. Most of the mathematical concepts and techniques will be presented in class, with plenty of opportunity for questions and clarification, but the best lessons learned are those derived from discussion and practice. Outside of class, it is essential that you read the assigned text sections, do the assigned homework, and bring any questions to class or to office hours. Mathematics is not a spectator sport. Don't just crank through computations – think about the problems posed, your strategy, the meaning of the computations you perform, and the answers you get. This will be the best preparation for interaction in the classroom and for the exams.

Homework: There will be one or two homework assignments corresponding to each chapter of the text. Homework assignments and due dates will be posted in the **Calendar** section of the course website. All homework assignments will be graded – primarily to ensure that the assignment was actually done, but with additional value for a job well done. You are encouraged to discuss the homework with your fellow students, but you must write up the solutions by yourself without collaboration with others. (This is simply a matter of professional ethics.) Homework assignments should be turned in either in class or slipped under the door of my office (Goldsmith 222) on or before the due date. Late homework will only be accepted with prior consent. All homework must be neat, with answers boxed when appropriate. Multiple pages must be stapled together. Solutions will be posted on the course website as PDF documents soon after the due date.

The reading assigned with each homework is essential. Some topics not covered fully in class will be left to the reading and you will be expected to pick up those additional details. Questions on the homework and the reading may also be directed to me via e-mail.

Some of the homework problems will look different than problems discussed in class and in the text. This is not an accident. We want you to *think* about the material and learn to apply it in unfamiliar settings and interpret it in different ways. Only if you understand the material (as opposed to merely knowing it) will you be able to go beyond the information you are given.

Exams and Grading: There will be two in-class hour exams tentatively scheduled for Monday, October 8 and Monday, November 5. There will also be a Final Exam on a date to be determined at the end of the course. Consult the course **Calendar** for possible changes. Your course grade will be computed according to the following scheme, subject to minor modification:

.20(hour exam 1) + .20(hour exam 2) + .25(homework) + .35(final exam)

Use of Technology: In some of the homework problems you will be asked not to use any technology (calculators or software packages). If no restriction is made, you may use the form of technology of your choice. Calculators will be permitted on the exams, but not computers or any other aids. You will probably want to have access to some form of

technology. It will be especially helpful if you are familiar with the matrix operations on a hand-held calculator. An effort will be made to write the exams in such a way that all problems may be solved without technology.

Words of Caution and Advice: This course may prove to be more demanding than your previous mathematics courses. The weekly assignments may be somewhat time-consuming, so you should plan now to set aside regular hours to wrestle with them. It is highly improbable that you will do well in this course without working the homework assignments in a timely fashion. New material builds on old, so don't fall behind. If you find yourself falling behind, please contact me immediately so that options for assistance may be discussed.

When you are working on your assignments, keep in mind that your success in this course will require more than just memorizing formulas and "plugging in values." Numerical calculations are still important, but the key to success is understanding the underlying concepts and working enough problems so that you can employ them in any example thrown at you, especially homework and exam problems which may differ significantly from material discussed in class.

Chapter	**Topics** (some topics may be omitted, if necessary)	**Sections**
Chapter 1	Systems of linear equations and their solutions; coefficient matrices and augmented matrices; row reduction; reduced row-echelon form; consistent vs. inconsistent systems; rank of a matrix; parameterization of solutions; matrix form of a linear system.	1.1 - 1.3
Chapter 2	Linearity and linear transformations; matrix of a linear transformation; finding the matrix of a geometrically defined linear transformation; meaning of the columns of a matrix; matrix algebra; matrix products and the composition of linear transformations.	2.1 - 2.4
Chapter 3	Subspaces; span; linear independence; kernel and image of a linear transformation; dimension; rank and nullity; basis of a subspace; coordinates relative to a basis; matrix of a linear transformation relative to different bases; similarity of matrices.	3.1 - 3.4
Chapter 4	General linear spaces (vector spaces); examples of spaces of matrices, spaces of functions; matrix of a general linear transformation defined on a finite-dimensional linear space; isomorphisms.	4.1 - 4.3
Chapter 5	Orthogonality; isometries; orthogonal matrices; orthonormal basis for a subspace; Gram-Schmidt process for producing an orthonormal basis from any basis; orthogonal projection; approximate (least squares) solutions; data fitting.	5.1 - 5.4
Chapter 6	Determinants; multilinearity property of determinants; volume of a k-dimensional parallelepiped.	6.1 - 6.3
Chapter 7	Invariant subspaces; eigenvalues and eigenvectors; algebraic and geometric multiplicity of an eigenvalue; diagonalization; discrete dynamical systems; diagonalization; powers of a matrix; stability; working with a) real eigenvalues, b) complex eigenvalues; and c) repeated eigenvalues.	7.1 - 7.6
Chapter 8	Spectral Theorem; symmetric matrices; quadratic forms.	8.1 - 8.2
Chapter 9	Continuous dynamical systems; systems of (1st order) linear differential equations; evolution matrices; stability of an equilibrium.	9.1 - 9.2
Supplements, etcetera	Nonlinear continuous dynamical systems; equilibrium analysis; phase-plane analysis.	9.3 and supplements